Math879, HW7

37. Let R be a rectangle which is tessellated by squares. Show that the ratio between its sides is rational.

38. Assume that $S = \{x_1, \dots, x_{57}\}$ is a set of 57 real numbers such that for any $1 \le i \le 57$ one can divide $S \setminus \{x_i\}$ into two sets A_i and B_i such that $|A_i| = |B_i| = 28$ and $\sum_{x \in A_i} x = \sum_{x \in B_i} x$. Prove that all x_i are equal.

39. Countably many people stand in a line so that the n-th one sees those numbered $n+1, n+2, \dots$. Everybody in the line wears a hat, which can be either white or black. The n-th man sees hats $n+1, n+2, \dots$, but he does not know the colors of hats $1, \dots, n$, including his own. Based on this information each of them has to guess what is the color of his own hat. Does there exist a strategy that allows them to make finitely many mistakes in any case?