Math879, HW6

31. Assume that X, Y, Z are three spheres touching each other from outside (i.e. neither of them lies inside the other one). Assume that $S_1, S_2, \ldots, S_7$ are seven spheres such that each S_i touches X, Y and Z from outside and S_i touches S_{i+1} from outside whenever $1 \leq i \leq 6$. Prove that $S_1 = S_7$.

32. Let $f \colon \mathbf{R}^2 \to \mathbf{R}$ be a continuous function with compact support. Assume that the integral of f over every line in the plan is 0. Show that $f = 0$.

33. Show that $\lim_{n \to \infty}(e^{-n} \sum_{k=0}^n n^k/k!) = 1/2$.

34. Let P be a set of (distinct) subsets of $\{1, \ldots, n\}$ such that $|P| > \binom{n}{n/2}$. Prove that there exists two distinct sets $X, Y \in P$ such that $X \subset Y$.

35. Let G be the group of all order preserving homeomorphisms from the unit interval to itself. Describe all the normal subgroups of G.

36. Let $p > 2$ be a prime and $\xi \neq 1$ a p-th root of unity. Set

$$S = \sum_{i=1}^{p-1} \left(\frac{i}{p}\right) \xi^i,$$

where $\left(\frac{i}{p}\right) = 1$ if i is a square modulo p and $\left(\frac{i}{p}\right) = -1$ otherwise. Prove that $S^2 = \pm p$ and find the sign.