Math879, HW5

25. Let P be a polygon with n vertices $P_1, \ldots, P_n, P_{n+1} = P_1$.

(a) Assume that one is given midpoints of the edges of P. Can one reconstruct P? Does the answer depend on n?

(b) Assume that one is given a triangle ABC and points $Q_1, \ldots, Q_n$ such that the triangles $P_i Q_i P_{i+1}$ (with this order) are similar to ABC. Can one reconstruct P?

26. Let $f, g \in \mathbf{Z}[x]$ be two relatively prime polynomials with $\deg(f) > \deg(g)$. Show that for every sufficiently large prime p the polynomial $pf(x) + g(x)$ is irreducible.

27. A map $f\colon X \to X$, where X is a compact metric space, is called distal if for any $x \neq y \in X$ one has that $\inf_{n \in \mathbf{N}} d(f^n(x), f^n(y)) > 0$. Show that a continuous distal map is surjective.

28. Construct a perfect (i.e. closed without isolated points) subset of $\mathbf{R}$ which does not contain any rational point.

29. Given points A and B in the plane and using compass only find points $P_0 = A, P_1, \ldots, P_{n-1}, P_n = B$ that divide the interval AB into n equal parts.

30. Assume that $a, b > 1$ are integers with $(a, b) = 1$, and p is a prime dividing $a^2 - 2b^2$. Prove that p itself is of the form $m^2 - 2n^2$ for $m, n \in \mathbf{Z}$.