Math879, HW4

19. Let γ_1 and γ_2 be two circles in the plane with a non-empty intersection. Show that one can construct their centers using straightedge only.

20. Let L be a lattice in $\mathbf{R}^n$ whose fundamental box has volume one. Show that a symmetric (i.e. $M = -M$) convex set in $\mathbf{R}^n$ whose volume is larger than 2^n contains a nonzero point of L.

21. Let C_i, $i \in \{1, 2, 3\}$ be three disjoint circles in the plane so that neither of them encompasses another one. Let P_{ij} be the intersection of the two outside tangents to the circles C_i and C_j. Show that these three points lie on a line.

22. Is it always true that given two polygons P, P' in $\mathbf{R}^2$ of the same area S, one can find a finite set of polygons $P_1, \ldots, P_n$ of total area S such that both P and P' can be covered by $P_1, \ldots, P_n$?

23. Can a cube and a regular tetrahederon of equal volumes be cut using finitely many hyperplanes to produce the same sets of pieces?

24. Show that $\int_0^1 \frac{dx}{x^x} = \sum_{n=1}^{\infty} \frac{1}{n^n}$.