Math879, HW3

13. Let γ_1 and γ_2 be two concentric circles on the plane. Show that one can construct their joint center using straightedge only.

14. By a strip of width l we mean an (unbounded) subset of $\mathbf{R}^2$ lying between two parallel lines with distance l between them. Assume that a circle of radius r is covered by s strips of lengthes $l_1, \ldots, l_s$. Prove that $2r \leq l_1 + \cdots + l_s$.

15. Show that given a sequence of numbers of length $n^2 + 1$ there is a monotonic subsequence of length $n + 1$.

16. Show that given any collection of $n > 2$ points in the plane which are not all on one line there is a line passing through exactly two of them.

17. Compute $\sum_{n=1}^{\infty} \frac{1}{n^2}$, $\sum_{n=1}^{\infty} \frac{1}{n^4}$.

18. Show that a bounded function $f \colon \mathbf{Z}^2 \to \mathbf{R}$ which satisfies

$$f(x, y) = \frac{1}{4}\left(f(x+1, y) + f(x, y+1) + f(x-1, y) + f(x, y-1)\right)$$

is constant.