Math879, HW2

7. For a simplex S in $\mathbf{R}^3$ its perimeter $P(S)$ is the sum of lengthes of its six edges. Prove that if $S \subseteq S'$ are two simplices then $P(S) \leq \frac{4}{3}P(S')$.

8. Let $a_1, \ldots, a_n$ be complex numbers. Find all eigenvalues of the matrix

$$\begin{pmatrix} a_1 & a_2 & \ldots & a_n \\ a_2 & a_3 & \ldots & a_1 \\ \ldots & \ldots & \ldots & \ldots \\ a_n & a_1 & \ldots & a_{n-1} \end{pmatrix}$$

9. Let K be a compact convex subset in $\mathbf{R}^d$ with smooth boundary. Show that it can be covered by $d+1$ subsets whose diameters are strictly smaller than the diameter of K.

10. Show that $\mathbf{Z}$ is not a disjoint union of finitely many arithmetic sequences with distinct minimal gaps.

11. Given a circle γ and a point P outside of it, construct the tangents from P to γ using straightedge only.

12. Show that a finitely generated group having no proper subgroup of finite index cannot be embedded in $\mathrm{GL}(n; C)$.