

On exam you should remember proofs of the following 10 Theorems:

Theorem 1. A field extension L/K is finite if and only if it is algebraic and finitely generated.

Theorem 2. If a field K is finite or countable, then there exists an algebraic closure \overline{K} .

Theorem 3. The field \mathbf{C} is algebraically closed.

Theorem 4. For any prime p and $n \geq 1$ there exists a field \mathbf{F}_{p^n} with p^n elements and $G_{\mathbf{F}_{p^n}/\mathbf{F}_p}$ is a cyclic group of order n generated by the Frobenius.

Theorem 5. Any finite separable extension L/K has a primitive element.

Theorem 6. (Artin) If L is a field and $G \subseteq \text{Aut}(L)$ is a finite subgroup, then L is a finite Galois extension of $K = L^G$ and $G = G_{L/K}$.

Theorem 7. (Galois correspondence) If L/K is a finite Galois extension then there is a one-to-one correspondence between subgroups of $G_{L/K}$ and intermediate fields $K \subseteq F \subseteq L$ which reverses inclusions.

Theorem 8. (Gauss lemma) Any primitive polynomial $f(x) \in \mathbf{Z}[x]$ irreducible in $\mathbf{Z}[x]$ is also irreducible in $\mathbf{Q}[x]$.

Theorem 9. If $n \in K^\times$, $\mu_n \subset K$ and L/K is cyclic of degree n , then there exists $\alpha \in L$ such that $L = K(\alpha)$ and $\alpha^n \in K$.

Theorem 10. If $n \in K^\times$, then there exists a primitive n -th root of unity $\xi_n \in \overline{K}$, the extension $K(\xi_n)/K$ is Galois and there is an embedding $G_{K(\xi_n)/K} \hookrightarrow (\mathbf{Z}/n\mathbf{Z})^\times$.

Material you are expected to know, use and cite appropriately.

Algebraic extensions, finite extensions, degree of an extension and its multiplicativity, minimal polynomial of an algebraic element, simple extensions, the lifting lemma, existence and uniqueness of splitting fields and algebraic closures, normal extensions and L^{nor} , the separable degree $i_{L/K}$ and its multiplicativity, separable elements and extensions, Frobenius endomorphism, index of inseparability $[L : K]_i$ and an example of inseparable extensions, Galois extensions, Galois correspondence and the corollaries for composed extensions LF/K , basic examples (mainly for groups S_3 and D_4), abelian extensions, construction with compass and straight-edge, the group μ_n and primitive roots of unity, finite fields, their classification and Galois theory, Eisenstein criterion and Gauss lemma over \mathbf{Z} , cyclotomic extensions and polynomials and their Galois theory, cyclic extensions, solvable extensions and an example of insolvable ones, radical extensions and resolvability theorem of Galois, symmetric polynomials, transcendental extensions, transcendence basis and degree.