Compliments in Linear Algebra 80146

by Amit Novick

First compiled: 04/12/2016

- Fibonacci sequence:

  $x_0 = 0, x_1 = 1, x_n = x_{n-1} + x_{n-2}$

  $V = \{(x_0, x_1, ...) | \forall n \in \mathbb{N}(x_{n+2} = x_n + x_{n+1})\}$

  Proposition:

  V is a *vector − space* over the field $\mathbb{R}$.

  $V \cong \mathbb{R}^2$(isomorphism)

  There is a *linear map* as follows:

  $\begin{cases} 0, 1, 1, 2, 3, ... \\ 1, 0, 1, 1, 2, 3, ... \end{cases}$

  (e_1, e_2)

  Question: Is there a non-zero sequence which we can formulatively describe?

  Guess: Maybe we can find this to be a *geometric sequence*?

  $1, a, a^2, a^3, ...$

  $\forall n(a^{n+2} = a^{n+1} + a^n) \iff a^2 - a - a = 0$

  $\frac{1 \pm 5}{2} = a_1, a_2$

  Every sequence $(x_0, x_1, ...) \subset V$ is a *linear combination* of $x(1, a_1, a_1^2, ...)+ y(1, a_2, a_2^2, ...) = (x + y, xa_1 + ya_2, xa_1^2 + ya_2^2, ...)$

  $= (0, 1, ...)$

  $\frac{1+\sqrt{5}}{2} \approx \frac{1+2.23}{2} \approx 1.6$

  $\begin{cases} x = -y \\ x\frac{1+\sqrt{5}}{2} - x\frac{1-\sqrt{5}}{2} = 1 \end{cases}$

  $\frac{1-\sqrt{5}}{2} < 1$, $x = \frac{1}{\sqrt{5}}$

  $(c_0 = 0, c_1 = 1, c_2 = 1, c_3 = 2, ...)$

  $c_n = \frac{1}{\sqrt{5}} \cdot (\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}} \cdot (\frac{1-\sqrt{5}}{2})^n$ (the latter expression converges to 0)

- Examples of $Vector\ Spaces$:

  $Pol = \mathbb{R}[x] \subset Func(\mathbb{R}, \mathbb{R}) = \mathbb{R}^{\mathbb{R}}$

  Formally: $\mathbb{R}[x] = \{(x_0, x_1, ..., x_n) | \forall n \in \mathbb{N}, x_0, x_1, ..., x_n \in \mathbb{R}\}$

  $x_0 + x_1 t + x_2 t^2 + ... + x_n t^n$

  Mentions $formal\ series$, $\{(x_0, x_1, x_2, ..) | x \in \mathbb{R}\}$ is a *map* $\mathbb{N} \to \mathbb{R}$.

  $V \subset \mathbb{R}^{\mathbb{N}}$shall be defined by the equations:

  $$\begin{cases} x_2 = x_0 + x_1 \\ x_3 = x_1 + x_2 \\ . \\ . \\ . \end{cases}$$

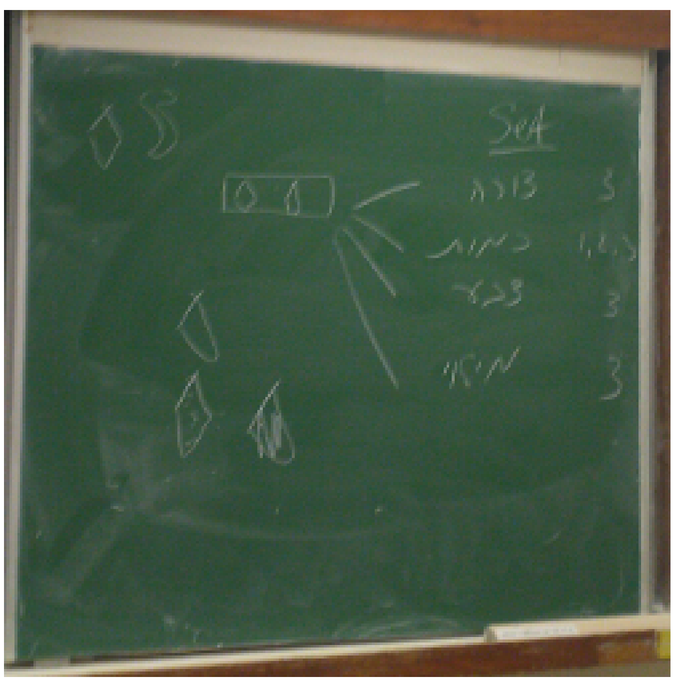
  In the course we shall see many more examples of $Vector\ Spaces$ $\{(x_1, ..., x_n)\} = \mathbb{R}^n$

  A *vector subspace* $\mathcal{U} \subseteq \mathbb{R}^n$ is given via the equations:

  $$\begin{cases} a_{11}x_1 + ... + a_{1n}x_n = 0 \\ . \\ . \end{cases}$$

- <u>Example from real life</u>:

  The game Set



Introduction:

In the game, a Set is a *set* of 3 cards, such that for every category of property (color,number,symbol,shading)

all members must either:

(a). share the same style of property

(b). be distinct from each other in style of property

For example, these three cards form a set:

- One red striped diamond
- Two red solid diamonds
- Three red open diamonds

In this example, shared properties were color and symbol, with styles red and diamond respectively,

while the distinct properties were number and shading, in which every card has distinct style.

Riddle: suppose all cards are spread out on the table, and somebody stole one card.

While looking at all the cards, is it possible to find out which card was stolen?

Idea: Define a $vector-space$ (or, more sufficiently, a $commutative-group$) on a set $S = (81 - cards)$,

with the $field$ $\mathbb{F} = \mathbb{F}_3$. (hint: $\mathbb{F}_3^4$), such that $\{(a, b, c) \in S^3\}$ is a Set $\iff a + b + c = 0$

Proposition: $\Sigma_{v \in S} v = 0$.

Corollary: The missing card equals $-\Sigma remainder$