Gauss elimination and chemistry.

Problem: balance the chemical reaction

$$\mathrm{xC_2H_5OH + yO_2 \to zCO_2 + tH_2O}$$

Solution: since the number of atoms of each kind before and after the reaction is the same, we obtain the following homogeneous system of linear equations

$$\begin{cases} 2x - z = 0 \\ 6x - 2t = 0 \\ x + 2y - 2z - t = 0 \end{cases}$$

The system is solved by Gauss method as follows:

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 1 & 2 & -2 & -1 & 0 \end{array}\right) \xrightarrow{R_{13}} \left(\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 0 \\ 6 & 0 & 0 & -2 & 0 \\ 2 & 0 & -1 & 0 & 0 \end{array}\right) \xrightarrow{R_2-6R_1;R_3-2R_1}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 0 \\ 0 & -12 & 12 & 4 & 0 \\ 0 & -4 & 3 & 2 & 0 \end{array}\right) \xrightarrow{-\frac{1}{12}R_2} \left(\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 0 \\ 0 & 1 & -1 & -\frac{1}{3} & 0 \\ 0 & -4 & 3 & 2 & 0 \end{array}\right) \xrightarrow{R_3+4R_2}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 0 \\ 0 & 1 & -1 & -\frac{1}{3} & 0 \\ 0 & 0 & -1 & \frac{2}{3} & 0 \end{array}\right) \xrightarrow{-R_3} \left(\begin{array}{cccc|c} 1 & 2 & -2 & -1 & 0 \\ 0 & 1 & -1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right) \xrightarrow{R_2+R_3;R_1+2R_3}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & -\frac{7}{3} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right) \xrightarrow{R_1-2R_2} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -\frac{2}{3} & 0 \end{array}\right)$$

The last matrix is of reduced row echelon form, and we obtain that the solution of the system of linear equations is $t = s$, $z = \frac{2}{3}s$, $y = s$, $x = \frac{1}{3}s$, where s is a parameter. Therefore the minimal solution with positive integral x, y, z and t is obtained for $s = 3$: $x = 1, y = 3, z = 2, t = 3$. Thus, the balanced reaction is

$$\mathrm{C_2H_5OH + 3O_2 \to 2CO_2 + 3H_2O}$$