

MATH 200 - SEC 201 - 2010W

Assignment no. 2

Due: 9am, Feb 11, 2011

1. Let

$$f(x, y) = \frac{xy^3 - x^3y}{x^2 + y^2}.$$

- (a) What is the domain of definition of f ?
- (b) Calculate f_x, f_y, f_{xy}, f_{yx} and verify that $f_{xy} = f_{yx}$.
- (c) Calculate $\lim_{x \rightarrow 0} f_{xy}(x, 0)$ and $\lim_{y \rightarrow 0} f_{xy}(0, y)$. Does $\lim_{(x,y) \rightarrow (0,0)} f_{xy}(x, y)$ exist?

2. Let

$$f(x, y) = x \arctan(x^2 - y).$$

- (a) What is the domain of definition of f ?
- (b) Calculate f_x and f_y and find the only point where the tangent plane is horizontal.
- (c) Find the second order Taylor approximation at the point from (b). Can you say whether this point is a local minimum, a local maximum or neither?

3. Let

$$f(x, y) = \sqrt{1 - x^2 - y^2}.$$

- (a) What is the domain of definition of f ?
- (b) Write the equation of the tangent plane at the point $(a, b, f(a, b))$ in terms of a and b .
- (c) Find a and b such that this tangent plane passes through the points $(1, 1, 1)$ and $(1, -2, 4)$.