

MATH 253 - SEC 104 - W2011T1

Quiz no. 1

2:00-2:20pm, Sep 20, 2011

Name (Last, First): _____ *Oni* _____

Student I.D. Number: _____

Signature: _____

INSTRUCTIONS: This is a closed-book exam. You may not use any books, notes, papers, calculators, or other aids. Do all work on the sheets provided. There is an extra sheet on the back for scratch work. If you need an extra sheet, raise your hand and one will be provided. If you need more space for your solution, use the back of the sheets and leave an arrow for the grader. Please draw a box around your final answer.

There are 4 questions, each worth 10 points. Explain all your answers. Good Luck!

1. Let $u = 2\vec{i} + \vec{j}$ and $v = 4\vec{i} + 8\vec{j} - 2\vec{k}$. Find a unit vector in the direction of $u - \frac{1}{2}v$.

$$u = \langle 2, 1, 0 \rangle$$

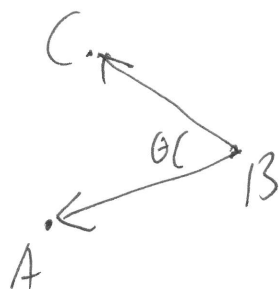
$$v = \langle 4, 8, -2 \rangle$$

$$w = u - \frac{1}{2}v = \langle 2 - \frac{4}{2}, 1 - \frac{8}{2}, 0 - \frac{-2}{2} \rangle = \langle 0, -3, 1 \rangle$$

$$|w| = \sqrt{0^2 + (-3)^2 + 1^2} = \sqrt{10}$$

$$\hat{w} = \langle 0, -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \rangle$$

2. Let $A = (3, 2, 0)$, $B = (1, 1, 6)$ and $C = (2, 5, 7)$. In the triangle ABC , is the angle $\angle ABC$, acute, right or obtuse?



$$\vec{BA} = \langle 3, 2, 0 \rangle - \langle 1, 1, 6 \rangle = \langle 2, 1, -6 \rangle$$

$$\vec{BC} = \langle 2, 5, 7 \rangle - \langle 1, 1, 6 \rangle = \langle 1, 4, 1 \rangle$$

$$\vec{BA} \cdot \vec{BC} = 2 \cdot 1 + 1 \cdot 4 + (-6) \cdot 1 = 0$$

$$\text{so } \cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

$\angle ABC = \text{right angle}$

3. Let u, v be two nonzero vectors, such that $\text{comp}_u v = \text{comp}_v u$. Is it necessarily true that $|u| = |v|$? Prove or give a counterexample.

$$\text{comp}_u v = \frac{u \cdot v}{|u|}$$

$$\text{comp}_v u = \frac{u \cdot v}{|v|}$$

$$\frac{u \cdot v}{|u|} = \frac{u \cdot v}{|v|} \Rightarrow |u| = |v| \quad \text{or} \quad u \cdot v = 0$$

$$u = \langle 1, 1 \rangle$$

$$v = \langle 1, -1 \rangle \text{ is a counterexample}$$

$$\langle 2, -2 \rangle$$

4. Consider the set of all points in the plane $P = (x, y)$ such that the distance from P to $A = (2, -2)$ is twice the distance from P to $B = (-4, 1)$. Show that this set is a circle and find its center and radius.

$$(x-2)^2 + (y+2)^2 = 4((x+4)^2 + (y-1)^2)$$

$$x^2 - 4x + 4 + y^2 + 4y + 4 = 4x^2 + 32x + 64 + 4y^2 - 8y + 4$$

$$0 = 3x^2 + 36x + 3y^2 - 12y + 60$$

$$x^2 + 12x + 36 + y^2 = 4y + 4 \quad \text{---} \quad 20 - 36 - 4 = 0$$

$$(x+6)^2 + (y-2)^2 = 20$$

center is $(-6, 2)$

radius is $\sqrt{20}$