Avoidance Coupling

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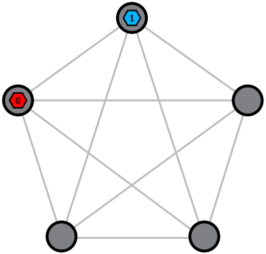

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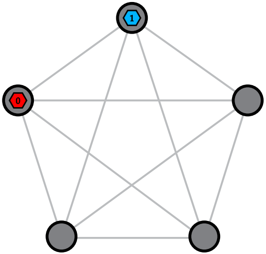

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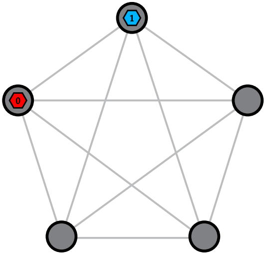

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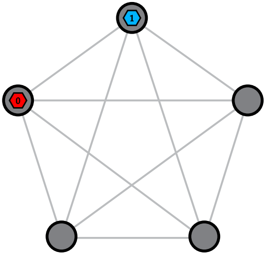

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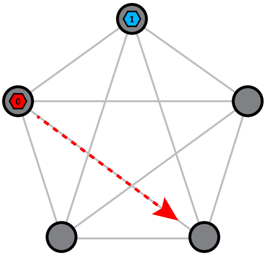

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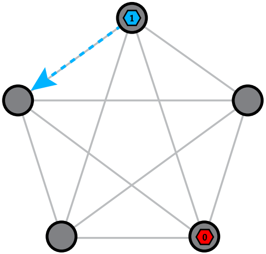

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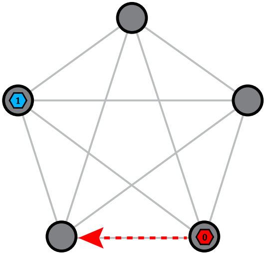

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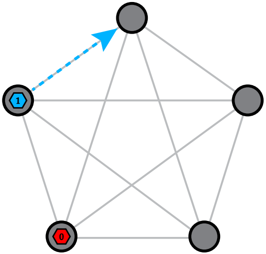

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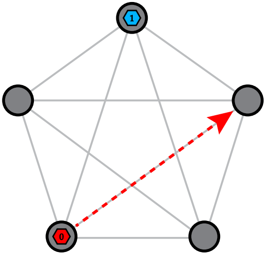

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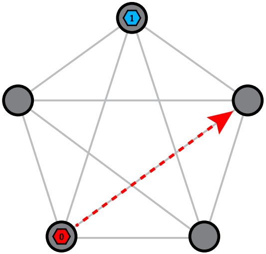

Q(AHMWW): Given G what is the maximal k for which an avoidance coupling exists?

Terminology

- Sites: $\subset \mathbb{Z}$
- Agents: $a_0, \ldots, a_{k-1}$.
- "Step": the movement of a single agent.
- "Round": the movement of all agnets.
- t: measures time in terms of rounds.
- K_n: complete graph.
- K_n^*: complete graph with loops.

Context and Motivation I

Coupling random walks

- Coupling of random variables $X_1, \ldots, X_k$ is their embedding in a joint probability space Ω.

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- SAC tends to be stronger, thus allows more agents.

Context and Motivation II

Relating a distribution and its marginals

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- Alexandrov's projection theorem (37'): one can reconstruct a convex body from all hyperplane projections.
  ...followed by an industry of obtaining useful information about convex bodies from various projected properties.
- k i.i.d. random walkers on a connected graph always collide.
  The contra-positive of our question is:

  When is it impossible for a joint distribution with the same marginals to avoid collision?

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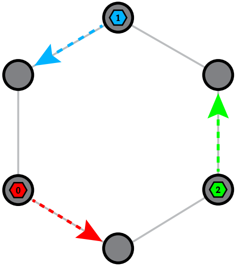
- Discrete time $\longleftrightarrow$ Continuous time poisson.
- In general starting position **cannot** be assumed uniform.

On a $2n$ cycle graph - maximal SAC is a least of size n.

- Lower bound: move all agents in the same direction in each round.



Simple Examples - I - Tori

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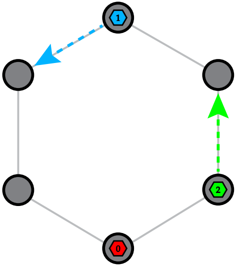

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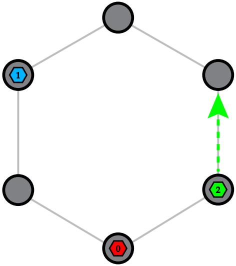

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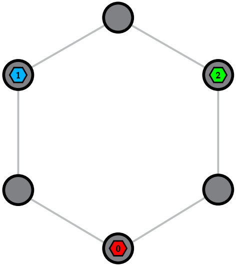

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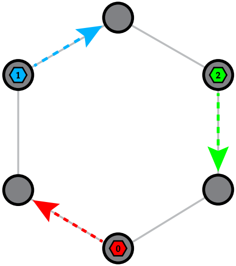

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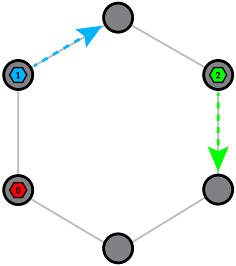

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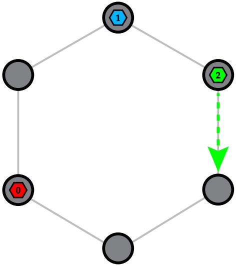

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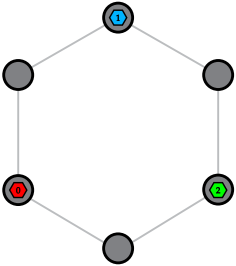

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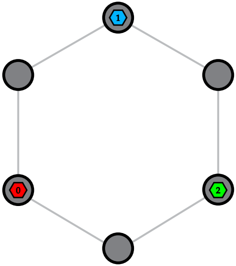

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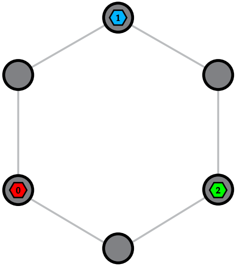

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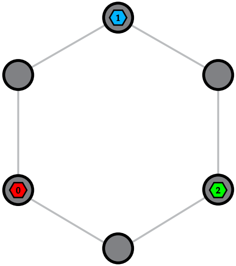
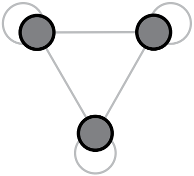
- The same principle works for $\mathbb{Z}^d/n\mathbb{Z}^d$,



Simple Examples - II - loop triangle

On a K_3^* - maximal SAC is of size 2.



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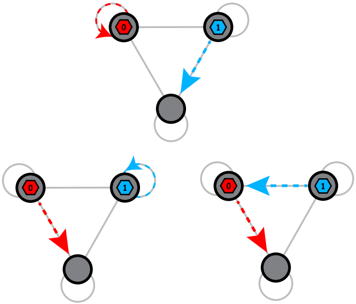

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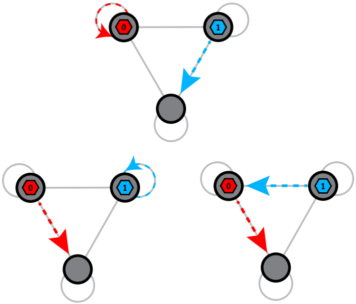
This walk is:

- minimum-entropy coupling,
- invariant to time reversal.



Results for K_n, K_n^*

Theorem (Angel, Holroyd, Martin, Wilson & Winkler)

Let $n = 2^{d+1}$. There exists a Markovian, minimum-entropy SAC of 2^d agents on K_n^*, K_{n+1}^* and K_{n+1}.

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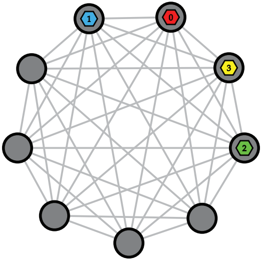
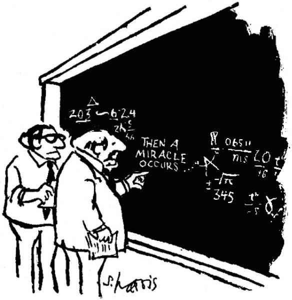
These couplings are hidden Markovian.

Constructing an Avoidance Coupling on K_{2^d+1}





"I think you should be more explicit here in step two."

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2^d agents SAC on $K_{2^{d+1}+1}$

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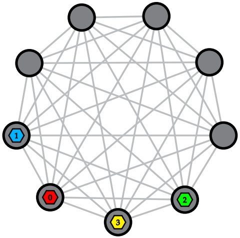

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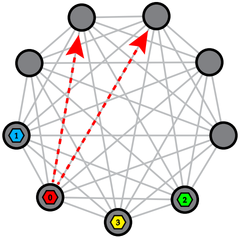

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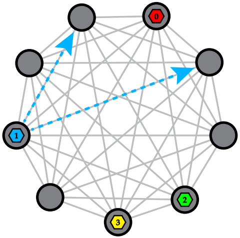

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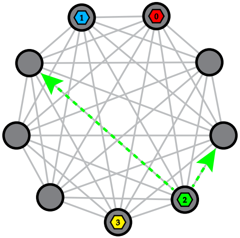

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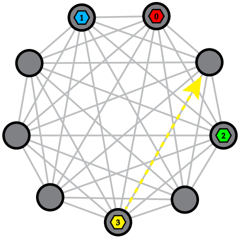
$a_3(t) = a_{11}(t)$ is fixed by $\varepsilon_t^0, \varepsilon_t^1$.



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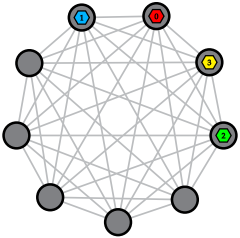
Let $\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ be i.i.d. uniform {-1,1} variables, and let δ_t be an independent uniform $\{0, 1\}$ variable. we set

$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

δ_t determines $a_0(t) = a_{00}(t)$.
Then ε_t^0 determines $a_1(t) = a_{01}(t)$,
and ε_t^1 determines $a_2(t) = a_{10}(t)$.
$a_3(t) = a_{11}(t)$ is fixed by $\varepsilon_t^0, \varepsilon_t^1$.



2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
$\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0,1\}$.

$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

We need to show:

- No collision in the same round
- Each agent performs simple random walk
- No collisions between rounds

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

We need to show:

- No collision in the same round - straightforward.
- Each agent performs simple random walk
- No collisions between rounds

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
$\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0, 1\}$.

$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

We need to show:

- No collision in the same round - straightforward.
- Each agent performs simple random walk - we show this first.
- No collisions between rounds

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
$\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0,1\}$.

$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

$a_m(t-1) \equiv a_n(t-2) + 2^d + \delta_{t-1} + \sum_{i=0}^{d-1} m^i \varepsilon_{t-1}^i 2^i,$

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

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$a_n(t-1) \equiv a_n(t-2) + 2^d + \delta_{t-1} + \sum_{i=0}^{d-1} 1 \cdot \varepsilon_{t-1}^i 2^i$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

$a_m(t-1) \equiv a_n(t-2) + 2^d + \delta_{t-1} + \sum_{i=0}^{d-1} m^i \varepsilon_{t-1}^i 2^i,$

$a_n(t-1) \equiv a_n(t-2) + 2^d + \delta_{t-1} + \sum_{i=0}^{d-1} 1 \cdot \varepsilon_{t-1}^i 2^i = 0,$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m. $\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0,1\}$.

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$a_n(t-1) \equiv a_n(t-2) + 2^d + \delta_{t-1} + \sum_{i=0}^{d-1} 1 \cdot \varepsilon_{t-1}^i 2^i = 0,$

$a_m(t-1) \equiv \sum_{i=0}^{d-1} (m^i - 1) \varepsilon_{t-1}^i 2^i.$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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$a_m(t-1) \equiv \sum_{i=0}^{d-1}(m^i - 1)\varepsilon_{t-1}^i 2^i$. Thus

$a_m(t) - a_m(t-1)$

$\equiv 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i + \sum_{i=1}^{d-1}(1 - m_{t-1}^i)\varepsilon_{t-1}^i 2^i$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \dots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m. $\varepsilon_t^0 \dots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0, 1\}$.

$$a_m(t) = 2^d + \delta_t + \sum_{i=0}^{d-1} m^i \varepsilon_t^i 2^i,$$

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$\equiv 2^d + \delta_t + \sum_{i=0}^{d-1} b^i(t) 2^i$ where b^i are i.i.d. Bernoulli $\{-1, 1\}$,

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

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$a_m(t) - a_m(t-1)$

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$\equiv 2^d + \delta_t + \sum_{i=0}^{d-1} b^i(t) 2^i$ where b^i are i.i.d. Bernoulli $\{-1,1\}$,

$\equiv \text{Unif}\{1 \dots 2^{d+1}\}$.

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
$\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0, 1\}$.

$$a_m(t) = 2^d + \delta_t + \sum_{i=1}^{d-1} m^i \varepsilon_t^i 2^i, \qquad a_m(t-1) \equiv \sum_{i=1}^{d-1} (m_i - 1)\varepsilon_{t-1}^i 2^i.$$

We need to show:

- No collision in the same round - Done.
- Each agent performs simple random walk - Done.
- No collisions between rounds

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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Let $m < q$ and recall that: $a_q(t-1) \equiv \sum_{i=1}^{d-1}(q_i - 1)\varepsilon_{t-1}^i 2^i$,

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

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and thus: $a_m(t) - a_q(t-1) \equiv 2^d + \delta_t + \sum_{i=1}^{d-1} \left[m^i \varepsilon_t^i + (1 - q^i)\varepsilon_{t-1}^i \right] 2^i$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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Write $\Delta^i := m^i \varepsilon_t^i + (1 - q^i)\varepsilon_{t-1}^i$.

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

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Write $\Delta^i := m^i \varepsilon_t^i + (1 - q^i)\varepsilon_{t-1}^i$. Taking $k = \max_i (m^i \neq q^i)$, we have

$m^k = 0$, $q^k = 1$,

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

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$$|\Delta^i| \leq \begin{cases} 1 & i > k \\ 0 & i = k \\ 2 & i < k \end{cases}$$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

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$$|\Delta^i| \leq \begin{cases} 1 & i > k \\ 0 & i = k \\ 2 & i < k \end{cases} \Rightarrow |\delta_t + \sum_{i=1}^{d-1} \Delta^i 2^i| < 1 + \sum_{i=2}^{d-1} 2^i < 2^d$$

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m. $\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0,1\}$.

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2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \dots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m.
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$\Rightarrow a_m(t) - a_q(t-1) \neq 0$.

2^d agents SAC on $K_{2^{d+1}+1}$ - cont.

$n = 2^d$, $V = \{0, \ldots, 2^{d+1}\}$, $a_n(t-1) = 0$. $m^i :=$ i-th binary digit of m. $\varepsilon_t^0 \ldots \varepsilon_t^{d-1}$ uniform {-1,1}, δ_t uniform $\{0,1\}$.

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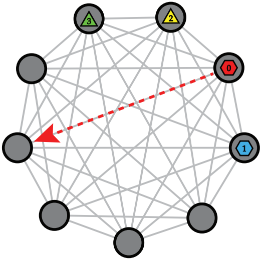
$\Rightarrow a_m(t) - a_q(t-1) \neq 0.$ □

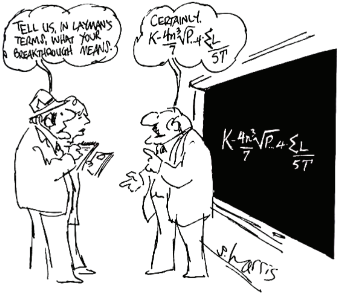
2^d agents SAC on K_{2^d+1} - cont.

And there is also an applet! (by David Wilson)

http://dbwilson.com/avoidance.svg

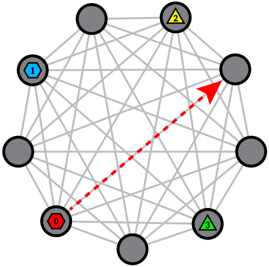
Monotonicity of Avoidance coupling on K_n





Partly Ordered Simple Avoidance Coupling (POSAC)

Let $G = (V, E)$ be a finite graph (loops and multi-edges are OK). m agents, $a_0, \ldots, a_{m-1}$ moving on V, are said to form a *k-POSAC* if:



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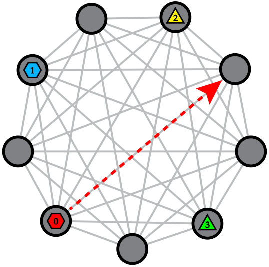
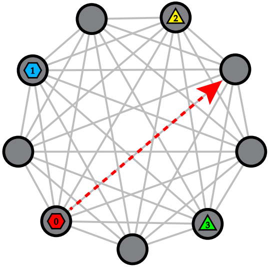
- The agents move one at a time **but in changing order,**



Partly Ordered Simple Avoidance Coupling (POSAC)

Let $G = (V, E)$ be a finite graph (loops and multi-edges are OK).
m agents, $a_0, \ldots, a_{m-1}$ moving on V, are said to form a k-*POSAC* if:

- The agents move one at a time **but in changing order,**
- **Agents** $a_1, \ldots, a_k$ **are always moving in order**,
- The agents never collide



Partly Ordered Simple Avoidance Coupling (POSAC)

Let $G = (V, E)$ be a finite graph (loops and multi-edges are OK). m agents, $a_0, \ldots, a_{m-1}$ moving on V, are said to form a *k-POSAC* if:

- The agents move one at a time **but in changing order,**
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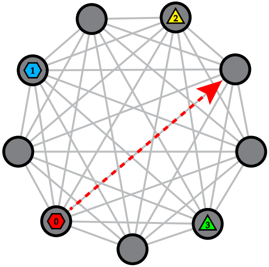

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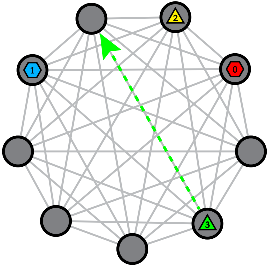

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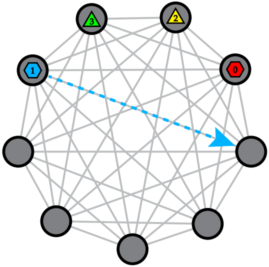

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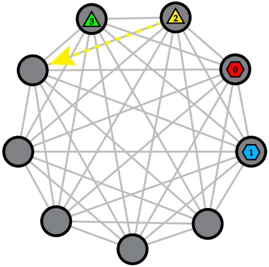

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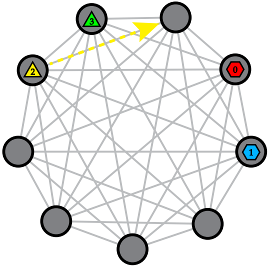

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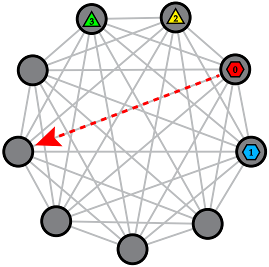
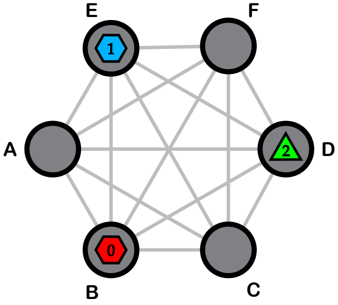
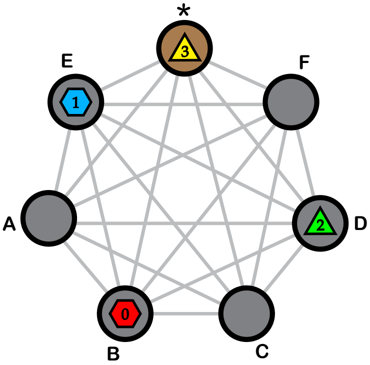
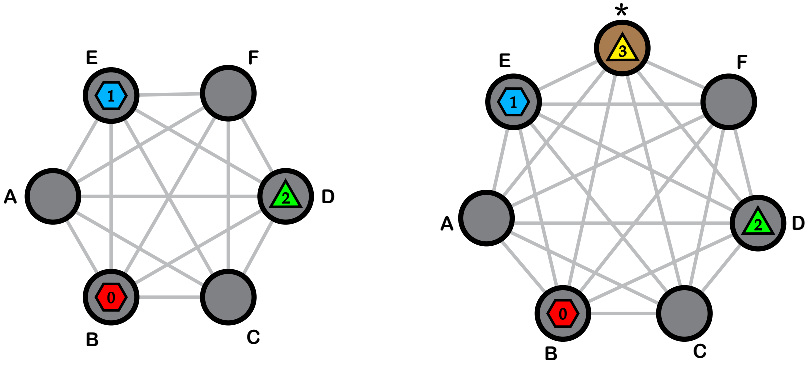




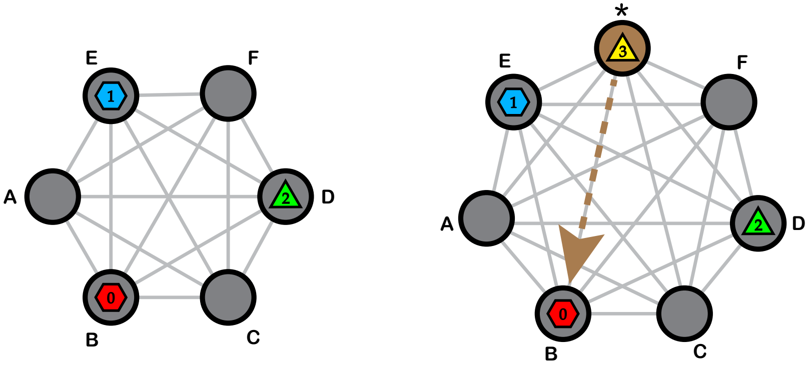


Theorem

If there is a k-POSAC of m agents on K_n, then there also is a k-POSAC of $m+1$ agents on K_{n+1}.

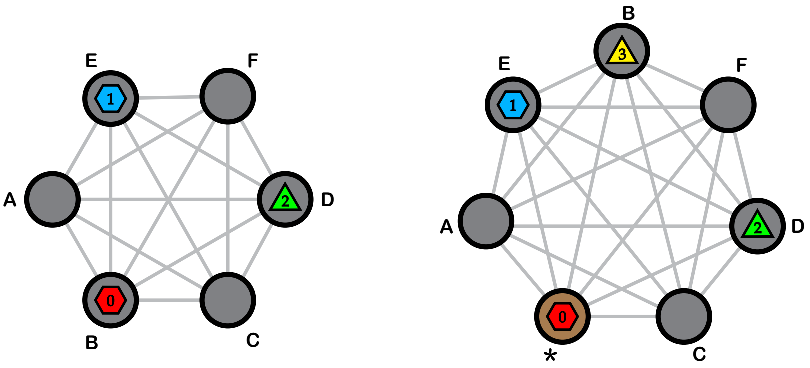


Add a special vertex $*$ with a special disordered agent.



Add a special vertex with a special disordered agent.

At start of a round flip the special vertex with another vertex.



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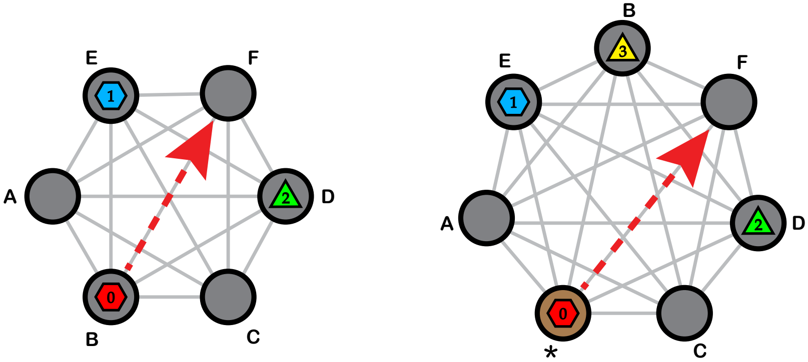

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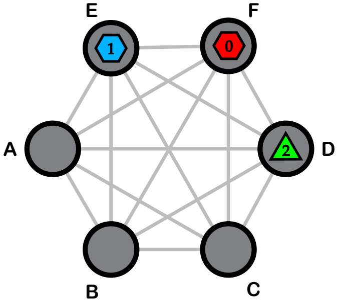

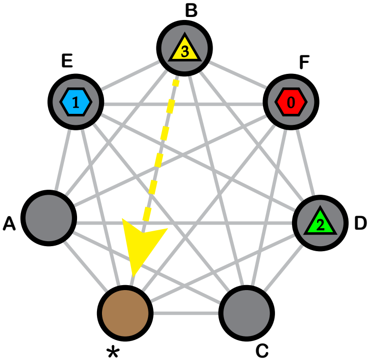

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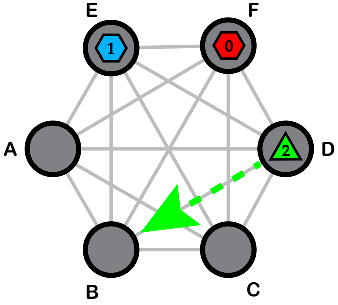

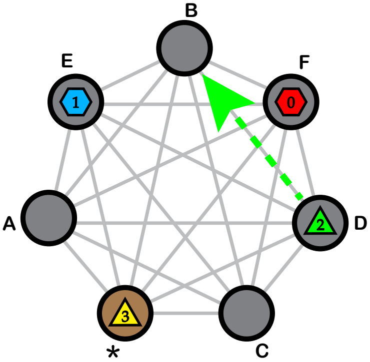

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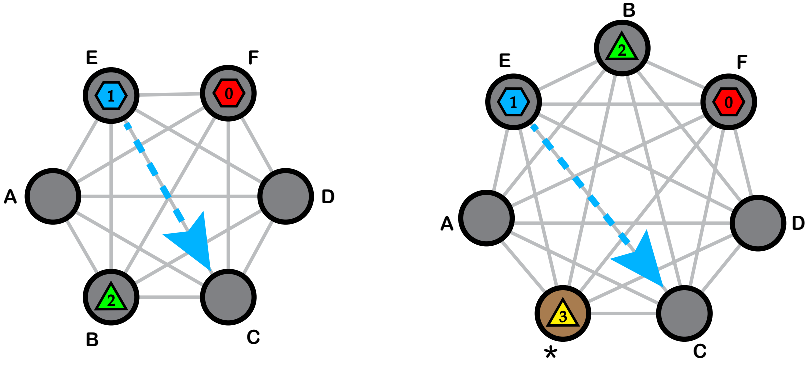

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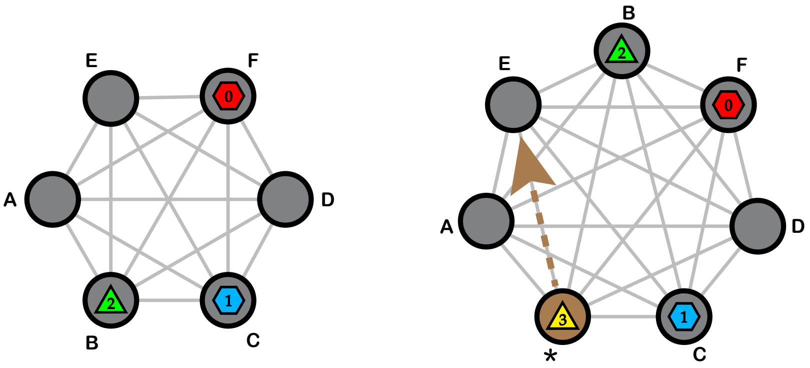

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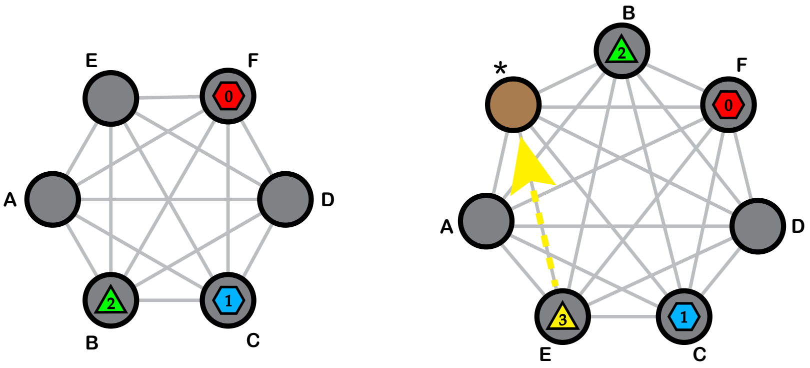

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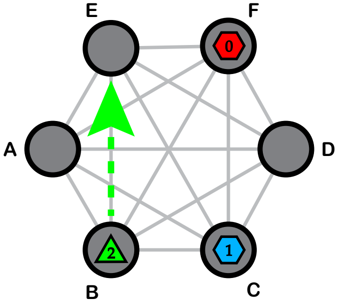

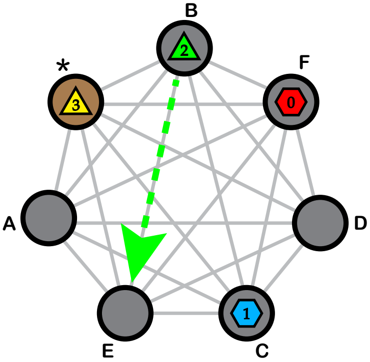

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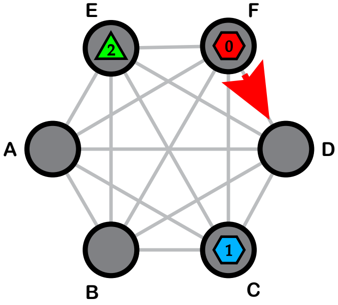

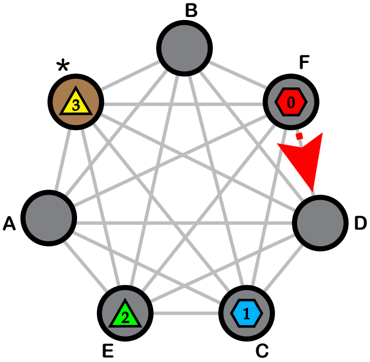

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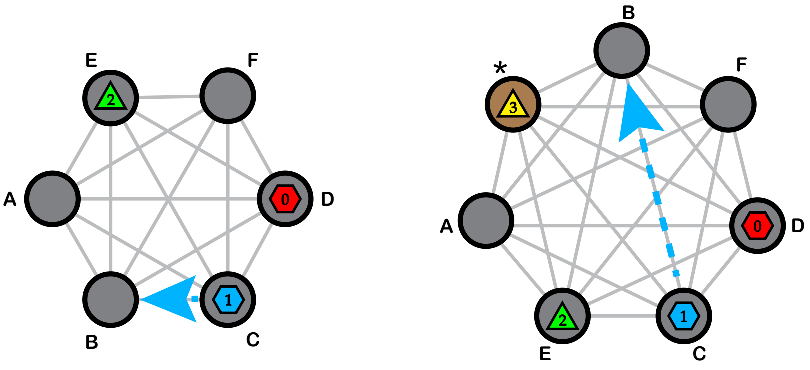

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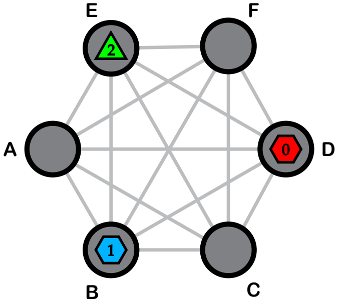

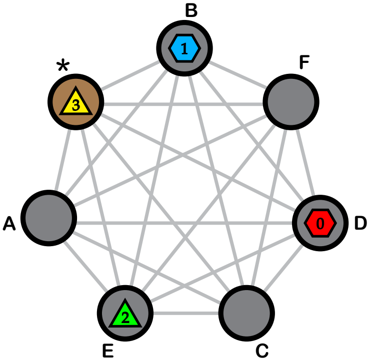

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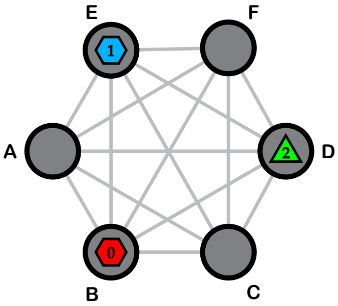
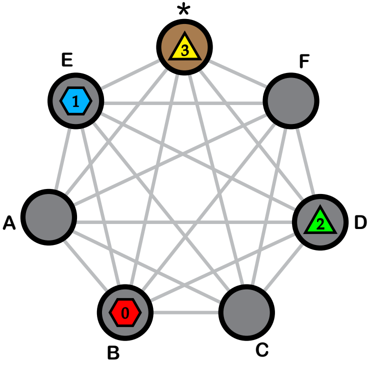
What is there to show?

- No collisions occur.
- Each walker makes a simple random walk.





What is there to show?

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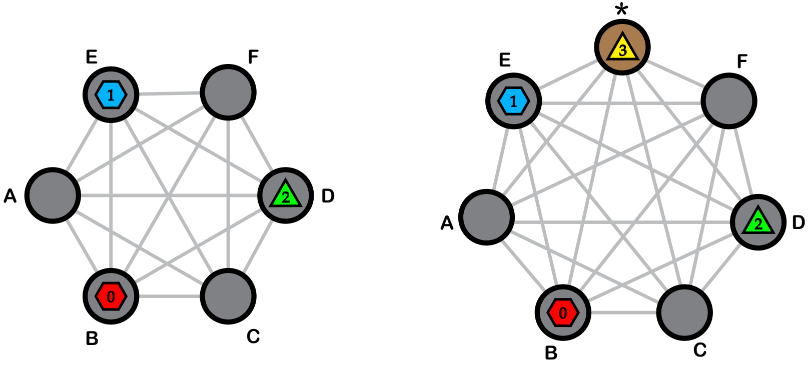

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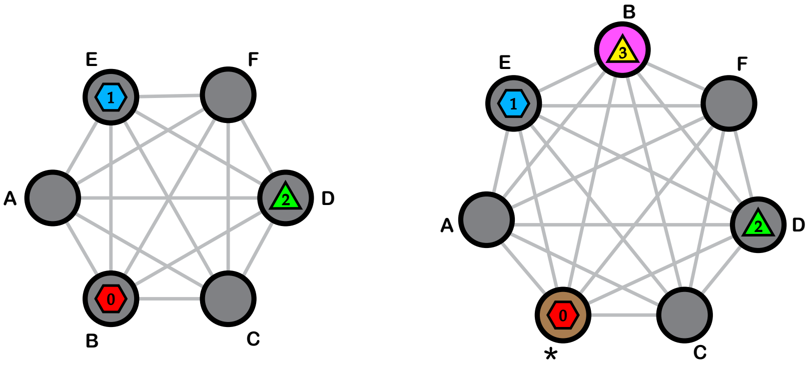

collusion can occur only in the previous $*$ vertex.

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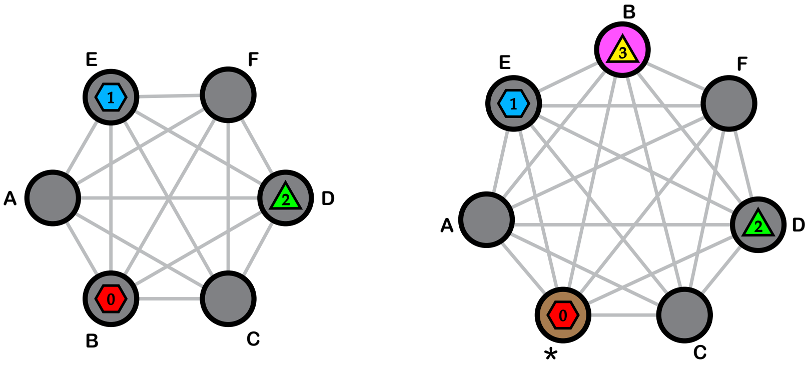
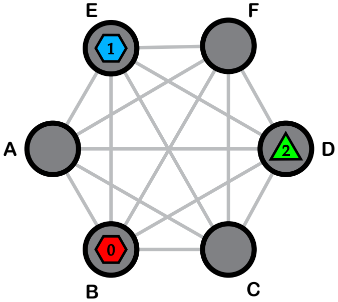
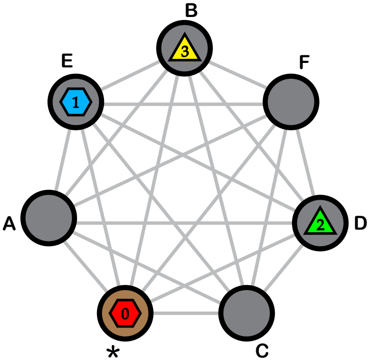


collusion can occur only in the previous $*$ vertex. However, it is occupied only as long as the new $*$ vertex is occupied.





Add a special vertex with a special disordered agent.

At start of a round flip the special vertex with another vertex.

Continue the process respecting the new labels.

As soon as $*$ clears - shift there the new agent.

What is there to show?

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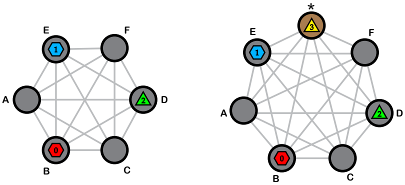

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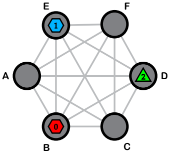

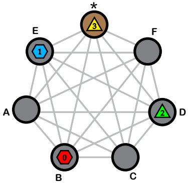

The new agent clearly makes a simple random walk.

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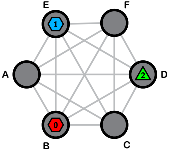

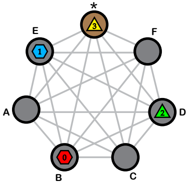

Other agents make a simple random walks on the $A - F$ labels.

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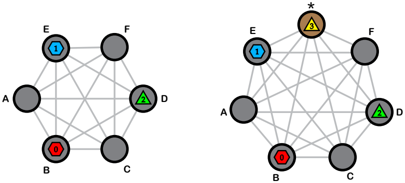

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What is there to show?

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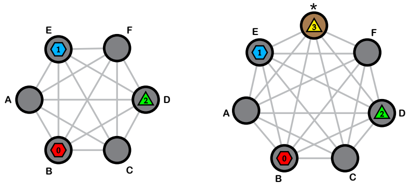

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$$\mathbb{P}(\text{it moved to that label}) \cdot \mathbb{P}(\text{the label isn't replaced by } *)$$

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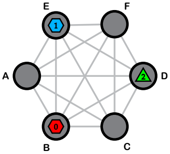

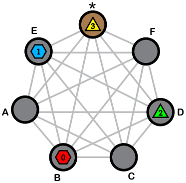

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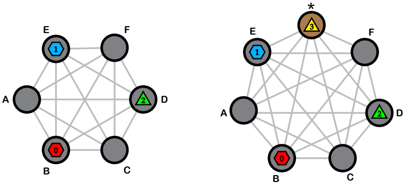

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The complementary $\frac{1}{n}$ is the probability of moving to the vertex currently labeled by $*$.

Open problems

- **Upper bound.**
- Is $\frac{K_n}{n} \to 1$?
- General & random graphs.
- High entropy avoidance coupling.

Thank you!

"How do you want it—the crystal mumbo-jumbo or statistical probability?"

* all cartoons by Sidney Harris.