Problems for M.Sc. Workshop no.9, December 23, 2012

Prof. Y.Kifer

53. A convex set C of area a and of perimeter $2p$ (i.e. $2p$ is the length of its boundary) is contained in a unit square S so that all points of C are father than $\epsilon > 0$ away from the boundary of S. A disk of radius ϵ is thrown randomly on S so that its center is uniformly distributed in S. Find the probability that this disk intersects C.

54. Prove that if $n \neq 2^k$ then any n real numbers can be reconstructed once we know their $n(n-1)/2$ pairwise sums. Show that for $n = 4$ this is not true, in general.

55. Let A be an $n-$element set and $A_1, ..., A_{n+1}$ be some of its nonempty subsets. Show that there exist two disjoint index sets $I, J \subset \{1, 2, ..., n+1\}$ such that $\cup_{i\in I} A_i = \cup_{j\in J} A_j$.

56. Let Q be the set of rational points. Prove that $Q \cap (0, 1)$ and $Q \cap [0, 1]$ are homeomorphic.

57. Given an integer $n \geq 3$ find all entire functions (i.e. analitic in the whole plane) f and g such that $f^n + g^n \equiv 1$.

58. Prove that in any separable metric space X there exists a set whose boundary contains all non isolated points of X.

59. Let $K_1, ..., K_n$ be some disks on the plane. Set $a_{ij} = area(K_i \cap K_j)$. Prove that $det(a_{ij}) \geq 0$.

60. Let $x_1, ..., x_n$ be nonzero vectors in a linear space and A be a linear transformation there such that $Ax_1 = x_1$, $Ax_k = x_k + x_{k-1}$, $k = 2, 3, ..., n$. Prove that $x_1, ..., x_n$ are linearly independent.