

**Problems for M.Sc. Workshop no.8, December 16, 2012**

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46. Show that any billiard trajectory in an ellipse is tangent to either an ellipse or a hyperbola with the same focal points.

47. Find the greatest common divisor of 1000th and 770th numbers in the Fibonacci sequence  $1, 1, 2, 3, 5, \dots$  (i.e.  $a_1 = a_2 = 1$ ,  $a_{n+2} = a_n + a_{n+1}$ ,  $n \geq 1$ ).

48. Let  $\xi_1, \xi_2, \dots$  be independent identically distributed random variables with a continuous distribution function. Set  $\nu = \min\{k : \xi_k > \xi_1\}$ . Find the distribution function and the expectation of  $\nu$ .

49. For any given  $n + 1$  parallel hyperplanes in  $R^n$  (i.e.  $n - 1$ -dimensional subspaces) show that there exists an equilateral simplex having vertices on these planes (one on each).

50. Is there a topological space  $X$  such that  $X \times X$  is homeomorphic to the real line  $R$ ?

51. Let  $f$  be a continuous linear functional on the Banach space  $c_0$  of converging to zero real sequences  $x = \{x_n\}$  with the norm  $\|x\| = \sup |x_n|$ . Prove that there exists a unique norm preserving extension of  $f$  to the space  $\ell_\infty$  of bounded real sequences.

52. Prove that among any 5 vectors in the Euclidean space one can choose two so that the length of their sum does not exceed the length of the sum of three remaining vectors.