Problems for M.Sc. Workshop no.6, November 25, 2012

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33. Suppose that each point of $\mathbb{Z}^2 \subset \mathbb{R}^2$ is a center of a disk of radius $\epsilon > 0$. Prove that any half line starting at $(0,0)$ intersects at least one of disks.

34. Let A and B be measurable sets on a circle $\mathbb{S}^1$ of length 1 with $\ell(A) = \alpha$ and $\ell(B) = \beta$ where ℓ denotes the Lebesgue measure. Show that some rotation of B intersects A by a set C with $\ell(C) \geq \alpha\beta$.

35. Find all Lebesgue integrable functions f on $[0,1]$ such that for any measurable subset $A \subset [0,1]$ of measure $1/2$ we have $\int_A f(x)dx = 1/2$.

36. Let μ be a probability measure on $\mathbb{R}^1$ and $d > 0$. Prove that the function $h(t) = |\int e^{itx} d\mu(x)|$ (where $i = \sqrt{-1}$) is $2\pi/d-$periodic if and only if for some a the measure μ is supported by the set of points $A_a = \{a + kd : k \in \mathbb{Z}\}$, i.e. $\mu(A_a) = 1$.

37. Let $f : K \to K$ be a map of a compact metric space K such that $f(K) = K$ and $\text{dist}(f(x), f(y)) \leq \text{dist}(x, y)$. Prove that f is an isometry.

38. Is it possible to approximate arbitrarily well the function z^{-4} on the circle $|z| = 1$ by (complex) polinomials there?