Problems for M.Sc. Workshop no.5, November 18, 2012
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27. Show that the decimal (fraction) $\alpha = 0.1234567891011...$ (all natural numbers are written successively) is an irrational number. Prove that fractional parts $\{10^n \alpha\}$, $n = 0, 1, 2, ...$ form a dense set on $[0, 1]$.

28. Disjoint disks of diameter 1 are put in the plane $\mathbb{R}^2$ so that every half line intersects at least one of them. Fix $x_0 \in \mathbb{R}^2$ and let A_n be a number of disks whose centers are within distance at most n from x_0. Prove that

$$\sum_{n=1}^{\infty} \frac{A_n}{n^2} = \infty.$$

29. Suppose that the sum $\xi + \eta$ of two independent integer valued random variables ξ and η has a binomial distribution. Prove that ξ and η also have binomial distributions.

30. Prove that no $n + 2$ vectors in $\mathbb{R}^n$ can have all pairwise angles larger than $\pi/2$.

31. Prove that there exists a real number α such that $1/3 \leq \{\alpha^n\} \leq 2/3$ for all natural n, where $\{a\}$ denotes the fractional part of a number a. Prove that the set of such α's have Lebesgue measure zero.

32. Is there exist a natural number n such that both 2^n and 5^n begin with the digit 5?