Problems for M.Sc. Workshop no.12, January 13, 2013

Prof. Y.Kifer

72. Let α be irrational and $\beta \in (0,1)$. Define $a_n(\alpha) = \min_{1\le m\le}\{m\alpha\}$ and $c_n(\alpha,\beta) = \min_{1\le m\le n}\{\beta - m\alpha\}$, $n = 1, 2, ...$ where $\{\cdot\}$ denotes the fractional part. Then there exists infinitely many n such that $a_n(\alpha) > c_n(\alpha, \beta)$.

73. Let $\xi = \sum_{n=1}^{\infty} 2^{-3^n}$. Then the inequality $|\xi - p/q| < cq^{-3}$ holds true for infinitely or finitely many natural numbers p, q if $c > 1$ or if $c = 1$, respectively.

74. Let Γ be a directed graph with N vertices such that any pair of vertices can be connected by a directed path in the graph. Let p(n) be the number of periodic paths in the graph with period not exceeding n. Prove that the limit $\lim_{n\to\infty} \frac{1}{n} \ln p(n)$ exists and express it via the spectral radius of the incidence matrix of Γ.

75. Let $\{x^{(k)}\}$ be a sequence of elements from the space l_1 such that $\lim_{k\to\infty} L(x^{(k)}) = 0$ for any linear continuous functional L on l_1. Prove that $\lim_{k\to\infty} \|x^{(k)}\|_{l_1} = 0$.

76. Find all real $n \times n$ nonnegative matrices having inverse matrix which is nonnegative, as well.