

**LECTURES ON MATHEMATICAL FINANCE AND RELATED
TOPICS
BOOK CORRECTIONS AND COMMENTS.**

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1. CORRECTIONS

- (1) p.104, 1.19 and 1.24: $(b - a)$ should be $(b - a)^{-1}$.
- (2) p.112, 1.7: $M_t^{(n)} = E(M_1^{(n)} | \mathcal{F}_t)$ (subindex t was missing).
- (3) p.112, 1.12: $\mathcal{M}_t^{(n)} = p_n^{(n)} M_t^{(n)} + \dots + p_{N_n}^{(n)} M_t^{(N_n)}$ (subindex t was missing).
- (4) p.134, 1.22: $\dots l = 0, 1, \dots, 2^n - 1, \dots$
- (5) p.137, 1.25: in the last line of (7.1.4) the last summand is $S_n(t_{l-1})S_n(t_{m-1})$, the subindex n of S is missing.
- (6) p.141, 1.5,8,9: E before exp should be deleted (since the expressions in the exponents are not random).
- (7) p.146, 1.12: $\in \mathcal{C}$ is missing, i.e. it should be $C = \cup_{j=1}^{\infty} C_j \in \mathcal{C}$.
- (8) p.153, 1.14: delete E (before $\int_0^s f(u)dW(u)$).
- (9) p.153, 1.25: replace \mathcal{F}_w by \mathcal{A}_w .
- (10) p.156, 1.20: write the factor 2^{2n} before $E \int_0^T (f_{n+1}(t) - f_n(t))^2 dt$.
- (11) p.157, 1.5: replace $f^{(n)}$ by f_n in both places.
- (12) p.157, 1.10: In fact, if (7.2.1) holds true and f_n is a sequence of simple functions such that $E \int_0^T (f(s) - f_n(s))^2 ds \rightarrow 0$ as $n \rightarrow \infty$, then f_n is a fundamental sequence in $L^2([0, T] \times \Omega, \ell \times P)$, where ℓ is the Lebesgue measure. By the Itô isometry $\int_0^t f_n(s)dW(s)$ is a fundamental sequence in $L^2(\Omega, P)$ for each $t \in [0, T]$, and so it converges in L^2 and its limit must be $\int_0^t f(s)dW(s)$ since we showed that along a subsequence it converges to this stochastic integral. In this sense the construction of stochastic integrals does not depend on approximating sequences of simple functions.
- (13) p.158, 1.22: replace $[2nt]$ by $[2^n t]$.
- (14) p.161, last line: replace 7.2.2(iii) by 7.2.2(ii).
- (15) p.165, 1.8 and 1.11: M_τ^2 and M_T^2 should be $M^2(\tau)$ and $M^2(T)$, respectively.
- (16) p.168, 1.27: it should be "... $I^M(\Phi_n)(t)$, $n \geq 1$ is also a Cauchy sequence ...".
- (17) p. 172, 2nd line in (7.3.3): in $\sum_{i,j=1}^d$ the upper limit d was missed.
- (18) p. 173, 1.16,17: employing the same arguments as at the end of Section 7.2.2 we can restrict ourselves to functions f and g satisfying $E \int_0^T f^2(s)ds < \infty$ and $E \int_0^T |g(s)|ds < \infty$, and so we approximate them by simple functions f_n and g_n having corresponding moments finite, as well.

- (19) p.180, 1.4: replace $\frac{\partial^2 F}{\partial x^2}(s, x)$ by $\frac{\partial^2 F}{\partial x^2}(s, X(s))$.
- (20) p.182, 1.7: local martingal ("local" is missed).
- (21) p.183, 1.26: in the 2nd line of (7.4.9) replace $Ee^{\frac{1}{2}M_{\varepsilon^2}(t)}$ by $Ee^{\frac{1}{2}\langle M_{\varepsilon^2} \rangle(t)}$.
- (22) p.183, last line: replace $EX_{\varepsilon}^r(T)$ by $EX_{\varepsilon}^r(T \wedge \tau_n)$.
- (23) p.184, 1.8: in the last line of the 4 lines formula replace $Ee^{\frac{1}{2}M_{\varepsilon^2}(T)}$ by $(Ee^{\frac{1}{2}M_{\varepsilon^2}(t)})^{1/q}$.
- (24) p.185, 1.4,6,7,8: replace $M(t)$ and $M(s)$ by $N(t)$ and $N(s)$, respectively, since M is reserved for the stochastic integral appearing in Corollary 7.4.1. On lines 9 and 10, M appears correctly.
- (25) p.190, 1.20: $R_n(t) = E \sup_{s \in [0, t]} |Y^{(n+1)}(s) - Y^{(n)}(s)|^2$, the square was missed.
- (26) p.190, the last expression in the 2nd line of the last formula: $4C^2 \int_0^t R_{n-1}(s) ds$, C^2 was missed.
- (27) p.191, 2nd line of (7.5.14): replace 32 by 64.
- (28) p.192, 1.11: replace $Y^{(k)}$ by $Y^{(k)}(s)$ in two places.
- (29) p.192, 1.25: $\int_0^t \sigma(s, Z(s)) dW(s)$, the integral limits (from 0 to t) were missed.
- (30) p.195, 1.4: replace σ_{ik} and σ_{jk} by $\sigma_{i,k}$ and $\sigma_{j,k}$, respectively.
- (31) p.195, last line: delete = 0.
- (32) p.197. 1.14: replace m by n i.e it should be $M_{t \wedge \tau_n}$.
- (33) p.197, 1.29: replace $\int_0^T \Phi(s) ds$ by $\int_0^T \Phi(s) dW(s)$.
- (34) p.198, 1.26,27: replace s by t in this two lines formula.
- (35) p.199, 1.9: replace $\frac{1}{2\sigma}$ by $\frac{1}{2\alpha}$ in the 2nd line of the formula.

2. COMMENTS

- (1) p.131: In Corollary 6.2.2 it suffices to assume (in addition to right continuity) that the processes Z_t and $-Y_t$ are left upper semi-continuous (i.e. that Y_t is left lower semi-continuous) which means that

$$\limsup_{s \uparrow t} Z_s \leq Z_t \quad \text{and} \quad \liminf_{s \uparrow t} Y_s \geq Y_t.$$

Then $R(\eta, t)$ is also left upper semi-continuous in t for any stopping time η . Since $\tau_{\zeta}^{\varepsilon} \uparrow \tau_{\zeta}^*$ as $\varepsilon \downarrow 0$ we obtain

$$\limsup_{\varepsilon \downarrow 0} E(R(\eta, \tau_{\zeta}^{\varepsilon}) | \mathcal{F}_{\zeta}) \leq E(\limsup_{\varepsilon \downarrow 0} R(\eta, \tau_{\zeta}^{\varepsilon}) | \mathcal{F}_{\zeta}) \leq E(R(\eta, \tau_{\zeta}^*) | \mathcal{F}_{\zeta}).$$

Since $Y_t \geq Z_t$ and Y_t is left lower semi-continuous, the function $R(t, \eta)$ is left lower semi-continuous in t for any stopping time η , i.e.

$$\liminf_{s \uparrow t} R(s, \eta) \geq R(s, \eta) \geq R(t, \eta),$$

and so

$$\liminf_{\varepsilon \downarrow 0} E(R(\sigma_{\zeta}^{\varepsilon}, \eta) | \mathcal{F}_{\zeta}) \geq E(\liminf_{\varepsilon \downarrow 0} R(\sigma_{\zeta}^{\varepsilon}, \eta) | \mathcal{F}_{\zeta}) \geq E(R(\sigma_{\zeta}^*, \eta) | \mathcal{F}_{\zeta}).$$

Since for any $\eta \in \mathcal{T}_{\zeta T}$,

$$\liminf_{\varepsilon \downarrow 0} E(R(\sigma_{\zeta}^{\varepsilon}, \eta) | \mathcal{F}_{\zeta}) \leq V_{\zeta} \leq \limsup_{\varepsilon \downarrow 0} E(R(\eta, \tau_{\zeta}^{\varepsilon}) | \mathcal{F}_{\zeta}),$$

we obtain

$$E(R(\sigma_{\zeta}^*, \eta) | \mathcal{F}_{\zeta}) \leq V_{\zeta} \leq E(R(\eta, \tau_{\zeta}^*) | \mathcal{F}_{\zeta}),$$

completing the proof.

- (2) p.214: In view of the above modification of Corollary 6.2.2 the conditions of Theorem 8.2.3 concerning the existence of a hedging investment strategy (π^*, σ^*) with the initial capital equal to the price V^* of a game option can also be relaxed so that the payoff processes $Y_t \geq Z_t$ should be càdlàg and in place of left continuity the processes Z_t and $-Y_t$ only need to be left upper semi-continuous.
- (3) p.290: Recently Yan Dolinsky constructed an example showing that in general there exists no shortfall risk minimizing strategy for continuous time Israeli contingent claims in a Black–Scholes frictionless market (see Y. Dolinsky, On shortfall risk minimization for game options, arXiv: 2002.01528).