For any root $\alpha \in R$ the imbedding $i_\alpha : sl_2(k) \to \mathfrak{g}$ defines a representation $\rho_\alpha : sl_2(k) \to End(\mathfrak{g})$ where $\rho_\alpha := ad \circ i_\alpha$ and therefore [see Problem 9 c) in Lecture 4] a representation $\tilde{\rho}_\alpha : SL(2,k) \to Aut(\mathfrak{g})$. We define

$$S_\alpha := \tilde{\rho}_\alpha(\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}) \in Aut(\mathfrak{g})$$

and denote by $S_\alpha(h)^\vee \in Aut(\mathfrak{g}^\vee)$ the corresponding automorphism of the dual space $\mathfrak{g}^\vee$. [If $\kappa : \mathfrak{g} \to \mathfrak{g}^\vee$ is an isomorphism defined by a non-degenerate invariant form $(,) : \mathfrak{g} \times \mathfrak{g} \to k$ then $S_\alpha(h)^\vee := \kappa^{-1} \circ S_\alpha \circ \kappa$].

Problem 0.1. *Show that*

a) $S_\alpha(h_\alpha) = h_\alpha$.

b) $S_\alpha(h) = -h$ *if* $(h, h_\alpha) = 0$.

c) $S_\alpha(\mathfrak{h}) \subset \mathfrak{h}$ *and* $S_\alpha|_{\mathfrak{h}} = s_\alpha$.

The rest of homeworks are from the Kirillov's book.

Homeworks : 7.1-7.6 ; 7.15 ; 7.17