Let $\mathfrak{g}$ be a reductive Lie algebra, $\mathfrak{h} \subset \mathfrak{g}$ the Cartan subalgebra, $k[\mathfrak{g}^\vee], k[\mathfrak{h}^\vee]$ the rings of polynomial functions on $\mathfrak{h}$ and $\mathfrak{h}$ and $k[\mathfrak{g}^\vee]^{\mathfrak{g}} \subset k[\mathfrak{g}^\vee], k[\mathfrak{g}^\vee]^W \subset k[\mathfrak{h}^\vee]$ the subrings of invariant polynomial functions. The Chevalley theorem sais that the restriction $k[\mathfrak{g}^\vee] \to k[\mathfrak{h}^\vee]$ induces an isomorphism $\tilde{r} : k[\mathfrak{g}^\vee]^{\mathfrak{g}} \to k[\mathfrak{h}^\vee]^W$. It is easy to prove the injectivity of r.

In the case when $\mathfrak{g} = gl_n$ we have to show that for any S_n-invariant polynomial function f on $\mathfrak{h}$ there exists an ad-invariant polynomial function $\tilde{f}$ on $\mathfrak{g}$ such that $f = r(\tilde{f})$.

Let E, E' be the fields of fractions of the rings $k[\mathfrak{g}^\vee]^{\mathfrak{g}}$ and $k[\mathfrak{g}^\vee]^W$ in the case $\mathfrak{g} = gl_n$. The morphism $\tilde{r}$ induces a field homomorphism $r : E \to E'$. I'll show that [in the case when $\mathfrak{g} = gl_n$] the homomorphism r is surjective. That is I'll will show that for any S_n-invariant polynomial function f on $\mathfrak{h}$ there exists $\tilde{f} \in E$ such that $f = r(\tilde{f})$.

By the construction E is the field of conjugacy invariant rational functions on gl_n. We denote by $F \supset E$ the Galois extension corresponding to the polynomial $P_A(t) := det(A - tId) \in E[t]$. By the construction the roots $\lambda_i, 1 \leq i \leq n$ of the polynomial $P_A(t)$ are elements of the field F, $Gal(F/E) = S_n$ and $\sigma(\lambda_i) = \lambda_{\sigma(i)}, \sigma \in S_n, 1 \leq i \leq n$. If f is an S_n-invariant polynomial function on $\mathfrak{h}$ then $\tilde{f} := f(\lambda_i) \in F$ is $Gal(F/E)$-invariant. So [by the Galois theory] it lies in E. It is clear that $f = r(\tilde{f})$.