

Social choice theory

Topics in Discrete Mathematics: CS/Math 945

Instructor: Gil Kalai

1 SYLLABUS

Social choice theory studies the way individual preferences aggregate into the choice of the society. It is thus originated in economics and political science. When “individuals” refer to processors in a distributed computer or to agents on the INTERNET we quickly reach areas of interest to computer science.

Week I: Introduction

Beginning with Condorcet.

We will discuss two results of Condorcet as prototypes to the relevance of mathematics to social choice. The Condorcet’s paradox and Condorcet’s Jury theorem.

Rationality and choice for individual agents

We will discuss three models for individual behavior and the notion of “rationality” as perceived in social sciences. We will consider, preference relations, the notion of utility, and choice functions.

Voting methods

We will describe and discuss informally a few voting methods used for choosing an alternatives among several. Among them: Borda’s method, approval voting, Plurality method and more.

Week II: Social welfare functions and Arrow's theorem and other models for collective choice.

We carefully develop the notion of social welfare functions formulate Arrow's theorem and give two proofs for the theorem.

Week 3: Arrow's theorem, Black theorem and the Gibbard-Satterthwaite theorem

We will continue the discussion of topics related to Arrow's theorem and consider the positive results of Black on aggregation of single picked preferences and the Gibbard-Satterwhite theorem about manipulation in social choice functions.

Problem set I: Expected Utility theory

The first problem set and an additional Friday meeting regarding it will deal with the von Neyman-Morgenstern utility theory.

Week 4: Power:

We will discuss two notions of powers for voting methods, the Shapley-Shubik and the Banzhaf power indices.

Week 5 Aggregation of information

We discuss the notion of information aggregation and results related to Condorcet's Jury theorem. Among them an extension of the Jury theorem to general voting games and the Jury theorem for strategic voting.

Problem set II will deal mainly with various binary voting methods and their properties

Week 6: Learnability and Indeterminacy

We will discuss theorems by McGarvey, McKelvey, Saari and other regarding social indeterminacy. We will relate it to issues of learnability and testability of choice behavior.

Week 7: Noise sensitivity and noise stability The notion of noise

sensitivity and social chaos will be demonstrated and discussed.

Week 8. Combinatorial, probabilistic and Fourier-methods

We will get deeper into the mathematical methods required to prove some of the results already discussed.

Weeks 9-12: Students will present in groups papers they have read on the matter of the course, in addition to a short written report. Among the topics: Judgment aggregation; Majority is stablest, Weak indeterminacy, and more