Generation

Theorem: The subcategory

$$\tilde{\mathcal{A}} := q_* p^* \underline{\mathcal{A}} \otimes_R \mathbb{K}_A \subset q_* \mathcal{F}(X^n, D) \otimes_R \mathbb{K}_A$$

split-generates it (i.e., the lifts of L to X^n split-generate $\mathcal{F}(X^n, D)$).

To prove this we apply Abouzaid's criterion: if

$$OC : HH_\bullet(\tilde{\mathcal{A}}) \to QH^{\bullet+n}(X)$$

hits the unit, $\tilde{\mathcal{A}}$ split-generates.

$$OC : HH_\bullet(\tilde{\mathcal{A}}) \to QH^{\bullet+n}$$

is dual to

$$CO : QH^{n-\bullet}(X) \to HH^{n-\bullet}(\tilde{\mathcal{A}})$$

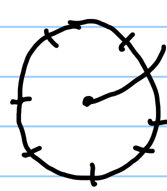
(as one can guess from the pictures; the identification $HH^\bullet \cong HH_\bullet^\vee$ requires a <u>weak CY structure</u> on Fuk).

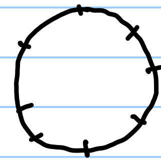
In particular, we find that

$$CO : QH^{2n} \to HH^{2n}(\tilde{\mathcal{A}})$$

is non-zero $\Rightarrow$ $\tilde{\mathcal{A}}$ split-generates.

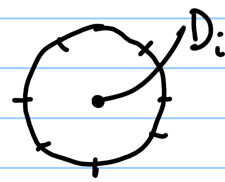
Lem: $CO([D_i]) = r_i \partial_{r_i}(\mu^0)$.

Pf: $CO([D_i])$ counts  D_i $\cdot\, r_1^{u\cdot D_1} \cdots r_k^{u\cdot D_k}$,

μ^0 counts  $\cdot\, r_1^{u\cdot D_1} \cdots r_k^{u\cdot D_k}$

$\ddots$

$$\Rightarrow r_i \partial_{r_i} \mu^0 = (u\cdot D_i)\cdot \circlearrowright$$

$$= \#$$ 

so they count the same thing.

Cor: $CO(\underset{\deg = 2n}{[D_i]^{*n}}) = [r_i \partial_{r_i} \mu^0]^{n}$ Yoneda product

So it suffices to check that

$$[r_i \partial_{r_i} \mu^0 \otimes 1]^n \neq 0$$

in $HH^{2n}(\tilde{\mathcal{A}})$.

We can compute $HH^0(\tilde{\mathcal{A}})$ directly: there is a Γ-action on $\tilde{\mathcal{A}}$ (covering group action), and

$$\sum_i r_i \partial_{r_i} \mu^0 \otimes 1 \in HH^2(\tilde{\mathcal{A}})^{\Gamma}$$

There is also a Γ^*-action on $\mathcal{A}$ (from the Γ-grading); and one can check that

$$HH^0(\mathcal{A})^{\Gamma} \cong HH^0(\tilde{\mathcal{A}})^{\Gamma^*} \text{ (as algebras)}$$

So it suffices to check it in $HH^0(\mathcal{A})^{\Gamma}$.

We consider