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Note Title 12/06/2015

Theorem

$$X^n = \left\{ \sum z_i^n = 0 \right\} \subset \mathbb{CP}^{n-1}$$

$$D \subset X^n, \quad D := \{ z_1 \cdots z_n = 0 \}$$

$$\tilde{Y}^n := \left\{ u_1 \cdots u_n + q \sum_i u_i^n = 0 \right\} \subset \mathbb{P}^{n-1}_{\mathbb{C}(q)}$$

$$G \circlearrowright \tilde{Y}^n \qquad\qquad \tilde{Y}^n / G$$

$$D^\pi \mathrm{Fuk}(X^n, D) \cong \psi^* D^b \mathrm{Coh}^G(\tilde{Y}_n) =: D^b \mathrm{Coh}\, \check{Y}^n$$

$\mathbb{C}((Q))$- linear

A_∞-quasi equivalence

$$\begin{tikzcd}
\mathcal{F}(X^n, D) \arrow[dd] & \supset \tilde{\mathcal{A}} \cong \psi^* \tilde{\mathcal{B}} \subset & \mathrm{Gr}\, MF^G(S,W) \cong D^b \mathrm{Coh}^G(\tilde{Y}^n) \arrow[d] \\
 & \mathcal{O}, \Omega^1(1), \dots, \Omega^n(n-1) \subset & \mathrm{Gr}\, MF(S,W) \cong D^b \mathrm{Coh}(\tilde{Y}^n) \arrow[d] \\
\mathcal{F}(\mathbb{CP}^{n-2}, D^n) & \supset \mathcal{A} \cong \psi^* \mathcal{B} \subset & MF(S,W)
\end{tikzcd}$$

$$\mathcal{A} \vee CF^*(L,L), \qquad \mathcal{B} = \mathrm{End}(k)$$

Remaining: prove $\tilde{\mathcal{A}}$ split generates $\mathcal{F}(X^n, D)$

Defn: We have maps

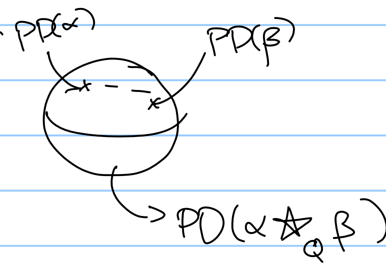
with Yoneda product

$$CO : QH^*(X) \longrightarrow HH^*(\mathrm{Fuk}(X,D))$$

$$OC : HH_*(\mathrm{Fuk}(X,D)) \to QH^{*+n}(X)$$

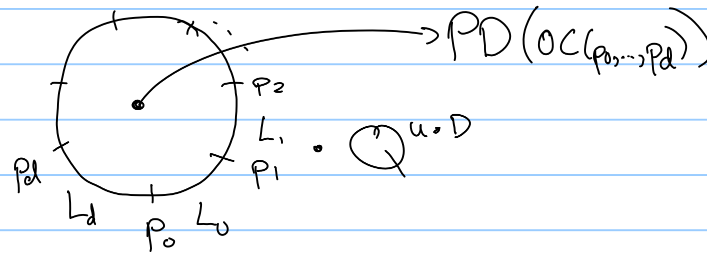
where $QH^*(X) := H^*(X; \mathbb{C}((Q)))$

$$\alpha \star_Q \beta := \sum$$



and CO is an algebra homomorphism.

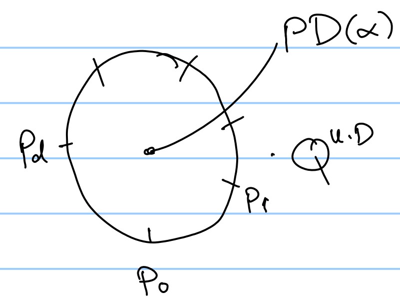
$OC(p_1 \otimes \cdots \otimes p_d) :=$



$CO(\alpha)(p_1 \otimes \cdots \otimes p_d) :=$

$\uparrow$

QH^*



Lem:

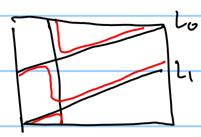
$$\begin{tikzcd}
QH^*(X) \arrow[r] \arrow[d, "CO"'] & QH^{2n-*}(X)^\vee \arrow[d, "OC^\vee"] \\
HH^*(\mathrm{Fuk}(X,D)) \arrow[r, phantom, "\cong"] & HH_{*+n}(\mathrm{Fuk}(X,D))^\vee
\end{tikzcd}$$

Theorem: If $\tilde{A} \subset \mathrm{Fuk}(X,D)$ full subcategory and

$$OC : HH_*(\tilde{A}) \longrightarrow QH^{*+n}(X)$$

hits $1 \in QH^0$, then $\tilde{A}$ split generates $\mathrm{Fuk}(X,D)$.

Motivation:



Cone - disconnected
one component is split generated by L_0, L_1
but not generated.

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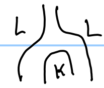
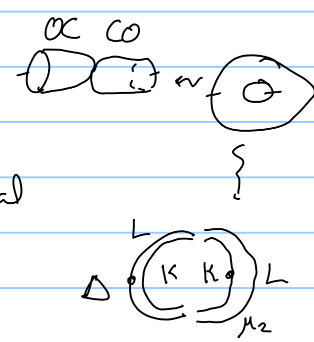
suppose hits 1

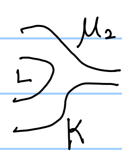
$$\begin{array}{ccc} HF^*(L,L) & \xrightarrow{OC} & QH^*(X) \\ \Big\downarrow \Delta & & \Big\downarrow CO \\ HF^*(K,L) \otimes HF^*(L,K) & \xrightarrow{\mu^2} & HF^*(K,K) \end{array}$$



unital



$p \qquad q$



$$K \underset{q}{\overset{p}{\rightleftarrows}} L$$

$\rightsquigarrow \ p \circ q = Id$

$OC : HH_{-n}(\tilde{A}) \longrightarrow QH^{0}(X)$

is dual to

$CO : QH^{2n}(X) \longrightarrow HH^{2n}(\tilde{A})$

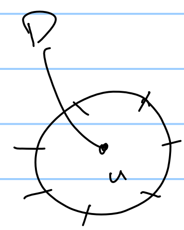
$\Longrightarrow$ suffices to show non-zero

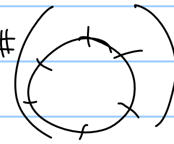
<u>Lemma</u>: $CO(PO([D_i])) = \left[Q_i \frac{\partial \mu^{\cdot}}{\partial Q_i}\right] \in HH^{2}(\tilde{A})$

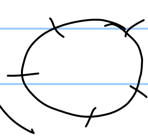
$\Big(\mu^{\cdot} \in CC^{0}(\tilde{A}), \quad [\mu^{\cdot}, \mu^{\cdot}] = 0 \Rightarrow Q\partial_Q [\mu^{\cdot}, \mu^{\cdot}] = 0$

$\Longrightarrow [\mu, Q\partial_Q \mu^{\cdot}] = 0$

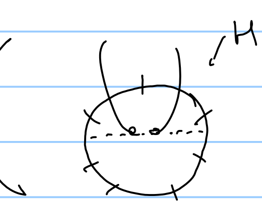
$\Longrightarrow$ cycle

<u>Pf</u>: 

$= u.D \cdot \#\Big($  $\Big) \cdot Q^{u \cdot D}$

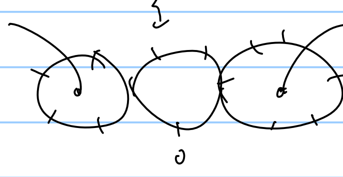
$= Q\partial_Q \Big($  $\cdot\, Q^{u.D}\Big)$

So, as CO is an algebra homomorphism,

$\Big($ 

$\rightsquigarrow$  $\Longrightarrow CO(\alpha \star_Q \beta) = CO(\alpha) \cup CO(\beta) + [d, H]$

$\wr$



$\Big)$

$\Longrightarrow$ it suffices to check that

$$\mathcal{CO}([D]^{\star n}) \neq 0$$
$$\parallel$$
$$[Q\partial_Q \mu^\circ]^n \neq 0$$

<u>Lem.</u> $[Q\partial_Q \mu^\circ]^n \neq 0$ in $HH^{2n}(\tilde{A})$

$\Longrightarrow$ $\tilde{A}$ split generates $\mathrm{Fuk}(X^n, D)$.

$\Longrightarrow$ proof of theorem complete.

What about $\mathcal{F}(X^n)$?

It's defined over

$$\Lambda := \left\{ \sum_i a_i Q^{\lambda_i} : \begin{array}{l} a_i \in \mathbb{C} \\ \lambda_i \in \mathbb{R},\ \lambda_i \to +\infty \end{array} \right\}$$

We should have $\mathcal{F}(X^n, D) \otimes_{\mathbb{C}((Q))} \Lambda \hookrightarrow \mathcal{F}(X^n)$

$L \subset X^n \setminus D$ exact $\qquad\qquad L \subset X^n$

Would like

①
$$\begin{tikzcd}
HH_*(\mathcal{F}(X^n, D) \otimes \Lambda) \arrow[r, hook] \arrow[d, "\mathcal{OC}"] & HH_*(\mathcal{F}(X^n)) \arrow[dl, "\mathcal{OC}"] \\
QH^*(X) &
\end{tikzcd}$$
?

(2) The split generation criterion should hold for $\mathcal{F}(X^n)$.

<u>Thm</u> (Perutz - S)

$$\begin{array}{ccc} \mathcal{F}(X,D) & & D^bCoh(Y) \\ \cup & & \cup \text{ split generates} \\ A & \cong & \mathcal{B} \end{array}$$

X, Y Calabi-Yau

Y has "maximally unipotent monodromy"

Then A split generates $\mathcal{F}(X,D)$