Nick III

Note Title 11/06/2015

<u>Recall</u>:

$$F(X^n, D) \cong p^* F(\mathbb{C}P^{n-2}, D^{-})$$

$$L : S^{n-2} \hookrightarrow \mathbb{C}P^{n-2} \setminus D$$

<u>Prop</u>: ① $CF^*(L,L) \cong HF^*(L,L)$

$$\cong \mathbb{C}[\theta_1, \dots, \theta_n]$$

(as assoc. alg)

② $\mu^n(\underline{z}, \dots, \underline{z}) = \pm z_1 \cdots z_n \cdot \mathbb{1}$

where $\underline{z} = \sum z_i \theta_i$

③ $\mu^1(\theta_i) = \pm Q_i \cdot \mathbb{1} + O(Q^2)$

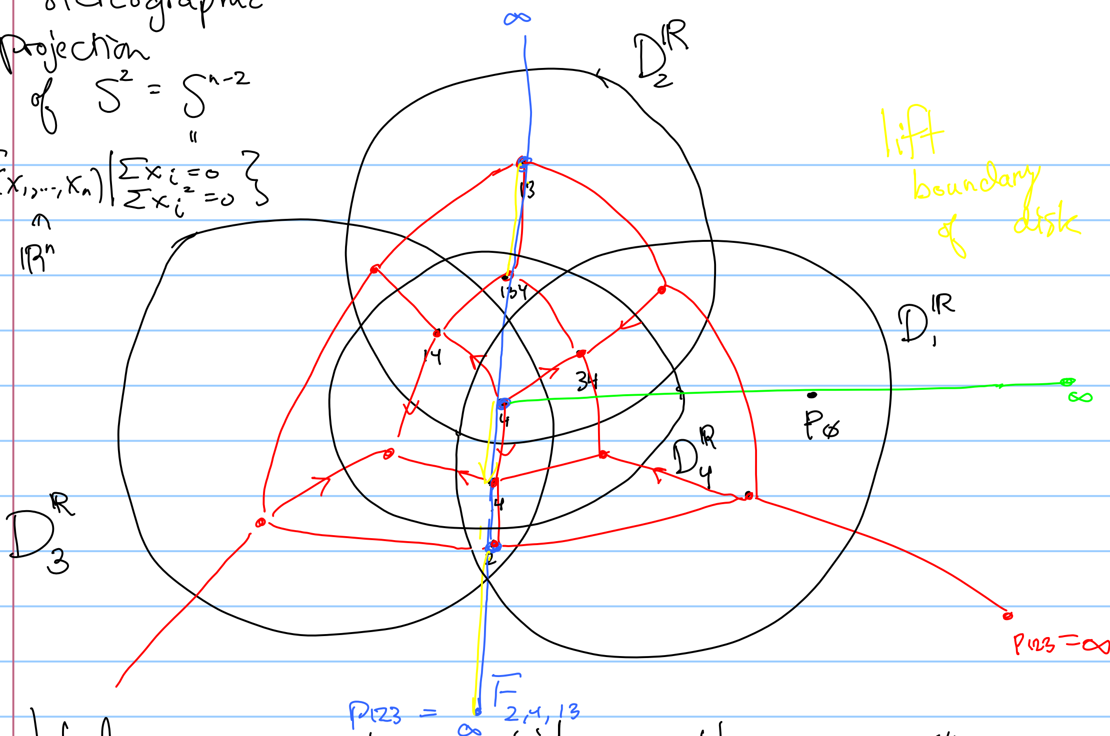
in $F(\mathbb{C}P^{n-2}, D)$

$$\begin{tikzcd}
L = \Gamma(\varepsilon df) \arrow[ddr, bend right] & D^*_\varepsilon(S^{n-2}) \arrow[d, "2:1"] \\
& D^*_\varepsilon \mathbb{R}P^{n-2} \arrow[d] \\
& \mathbb{C}P^{n-2}
\end{tikzcd}$$

with $\Gamma(\varepsilon df) \subset D^*_\varepsilon(S^{n-2})$

Stereographic Projection of $S^2 = S^{n-2}$

$$\{(x_1,\dots,x_n) \mid \textstyle\sum x_i = 0,\ \sum x_i^2 = 0\} \subset \mathbb{R}^n$$



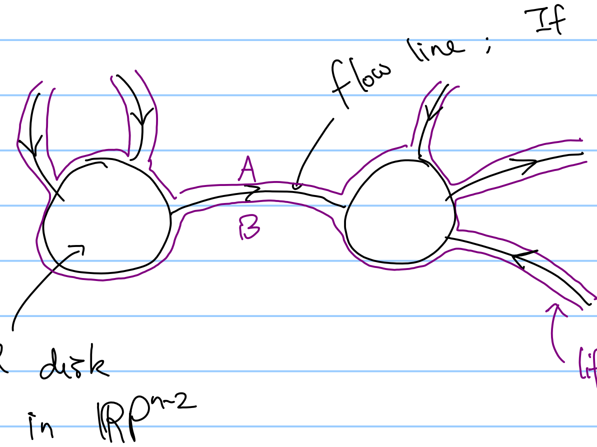
label region by which variables are positive in it

Grad flow lines of $f(x) := \sum_i g(x_i)$

Lim $\varepsilon \to 0$: count

If A,B lift to different sheets → flow of f

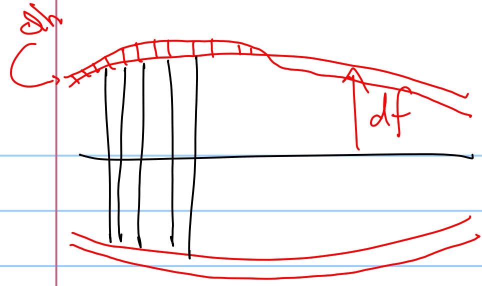
If A,B lift to same sheet, → flow of h



$$CF^*(L,L) \cong CM^*(f) \oplus CM^*(h)$$

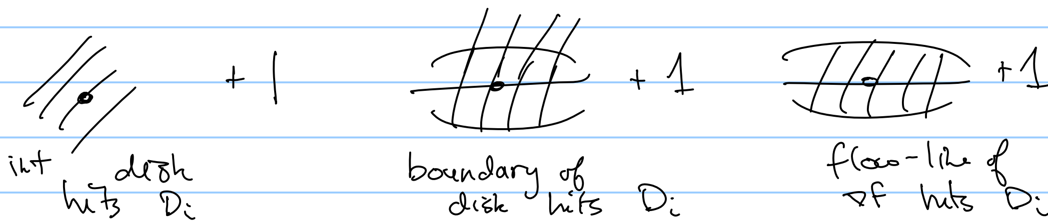
$$\mathbb{C}\cdot p_K : \quad K \subset \{1,\dots,n\}, \quad K \neq \phi, \{1,\dots,n\}$$

$$\mathbb{C}\cdot p_\phi,\ \mathbb{C}\cdot p_{\{1,\dots,n\}}$$

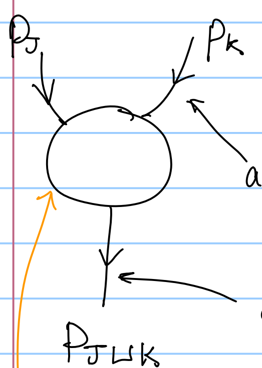


Picture to explain (*)

Also need int # with D_i to be 0:



Now compute coeff of $P_{J\sqcup K}$ in $\mu^2(P_J, P_K)$:



asc. mfld of P_K is open in F_K

desc mfld open in $F_{J\sqcup K}$

must be degree 1 disk, i.e. half real line

Defn: $F_{K_1,\dots,K_k} := \{x \in S^{n-2} : x_i = x_j \ \forall i,j \in K_\ell \ \forall \ell\}$

$\Rightarrow$ this line is $F_{J,K,\overline{J\sqcup K}}$

$\{(x_1,x_1,x_2,x_3) : \sum x_i = 0\}$

"

$F_{12,3,4}$

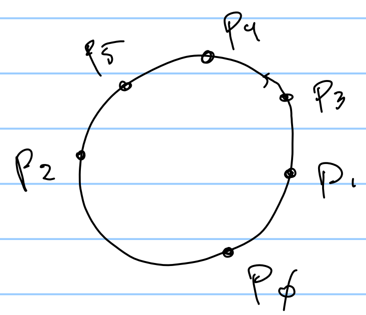
In picture of stereo projection, take $J=2$, $K=4$, $\overline{J\sqcup K} = 13$

$\mu^n(\underline{z},\dots,\underline{z}) = \pm z_1 \cdots z_n$

First, only $\mu^n(\Theta_{\sigma(1)}, \dots \Theta_{\sigma(n)})$ can be non-zero (grading in $H_1(\mathbb{C}P^{n-2})$)

(and output $= 1$)

To compute coefficient, count:

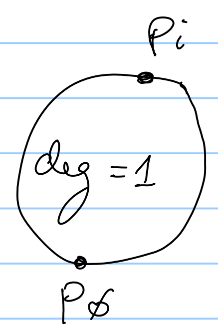


Veronese's thm: unique such curve of deg $= n-2$ through $n+1$ pts in $\mathbb{C}P^{n-2}$.

$\mu^1(\Theta_i) = \pm Q \cdot 1 + O(Q^2)$

Grading $\Rightarrow \mu^1(\Theta_i) = c \cdot Q_i \cdot 1 + O(Q^2)$

c counts



unique $\boxed{\text{real line}}$ through p_i, p_ϕ crosses $D_i^{\mathbb{R}}$ once negatively, all others positively

in picture

$\Rightarrow$ comes with coefficient Q_i