Nick II

Note Title 6/10/2015

Last talk:

$\pi : (Y,E) \longrightarrow (X,D)$ branched cover
(about E_i, with order a_i)

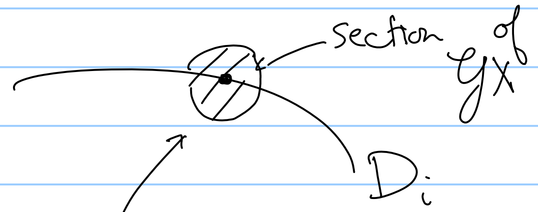
$$\Longrightarrow \text{ get } \begin{array}{ccc} \mathbb{Z} & \longrightarrow & H_1(\mathcal{G}(Y\setminus E)) \\ \| & & \big\downarrow \pi_* =: p \\ \mathbb{Z} & \longrightarrow & H_1(\mathcal{G}(X\setminus D)) \end{array}$$

and $\mathcal{F}(Y\setminus E) \cong p^* \mathcal{F}(X\setminus D)$ $\left(\cong \mathcal{F}(X\setminus D) \times \Gamma^*\right)$

but $\mathcal{F}(Y,E) \not\cong p^* \mathcal{F}(X,D)$

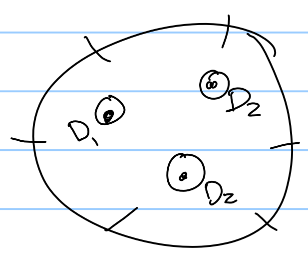
(1) Define $R := \mathbb{C}[[Q_1,\dots,Q_n]]$ to be
$H_1(\mathcal{G}(X\setminus D))$-graded

Q_i has degree =



restrict to
boundary $= \deg(Q_i) \in H_1(\mathcal{G}(X\setminus D))$

Then, $\mathcal{F}(X,D)$ is $H_1(\mathcal{G}(X\setminus D))$-graded as before
$\cong$ (if X CY)
$\mathbb{Z} \oplus H_1(X\setminus D)$



(2) Define $\mathcal{F}(X, D^{\underline{a}})$: everything the same as for $\mathcal{F}(X, D)$, except

we count discs

$$u : (D, \partial) \longrightarrow (X, L_i)$$

that are tangent to D_i to order $a_i - 1$ wherever they meet.

This defines an A_∞ cat., and

$$\mathcal{F}(Y, E) \simeq p^* \mathcal{F}(X, D^{\underline{a}})$$

$\Longrightarrow$ We want to compute in

$\mathcal{F}(\mathbb{C}P^{n-2}, D^{\underline{n}})$

(as we have
$$\begin{array}{c} (X^n, E) \\ \downarrow \\ (\mathbb{C}P^{n-2}, D) \end{array}$$
)

Prop. There exists a Lag. immersion

$L : S^{n-2} \looparrowright \mathbb{CP}^{n-2} \setminus D$

s.t.

(0) $CF^*(L,L)$ is defined as A_∞-alg!

(1) $CF^*(L,L) \cong HF^*(L,L)$ $(\mu^1 = 0)$

$$\cong \Lambda^* \mathbb{C}^n$$

$$\cong \mathbb{C}[\theta_1, \dots, \theta_n] \quad (\text{in } \mathcal{F}(\mathbb{CP}^{n-2} \setminus D))$$

as assoc. alg.

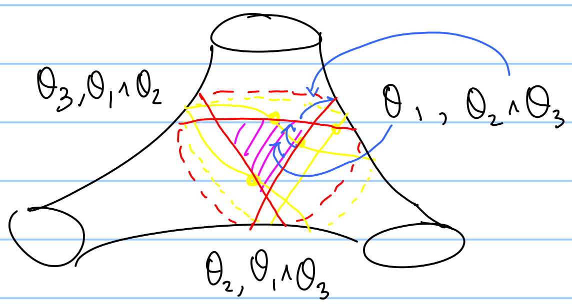
(2) $\mu^n(\underline{z}, \dots, \underline{z}) = \pm z_1 \cdots z_n$

where $\underline{z} := z_1\theta_1 + \dots + z_n\theta_n$

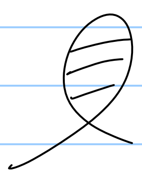
(3) In $\mathcal{F}(\mathbb{CP}^{n-2}, D)$,

$\mu^1(\theta_i) = \pm Q_i \cdot 1 + O(Q^2)$

$n = 3$



See two generators for each self-int by pushing L off itself by Ham isotopy.

Potential problem:  $\rightsquigarrow \mu_0$

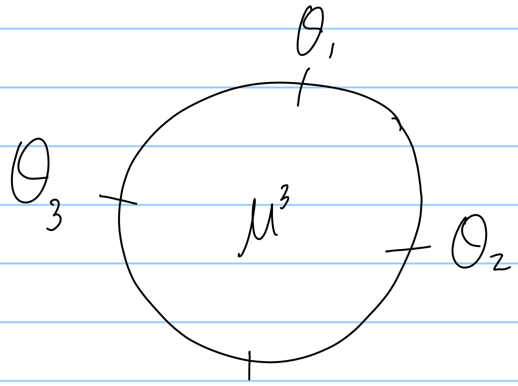
$$CF^*(L,L) \cong \underbrace{H^*(S^{n-2})}_{\mathbb{C}\cdot 1 \oplus \mathbb{C}\cdot \theta_1 \dots \theta_n} \oplus \quad \cdots$$

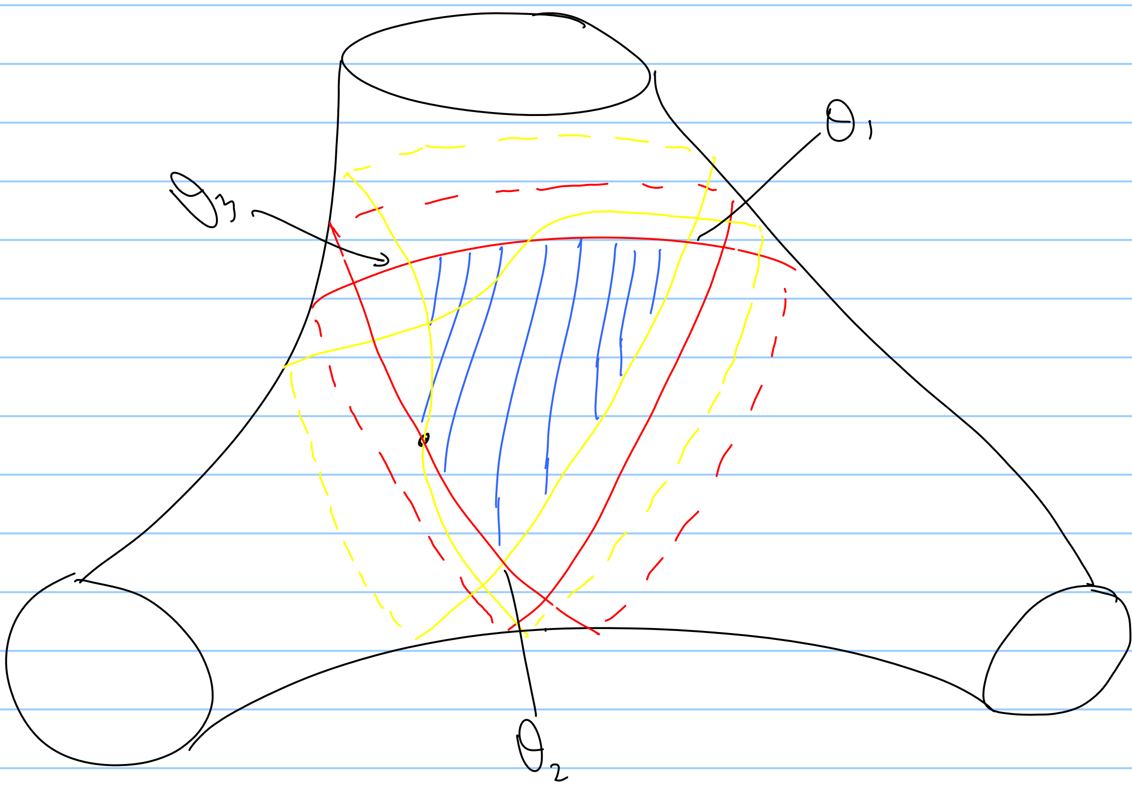
$$\cdots \oplus \quad \mathbb{C}\cdot (p,q)$$

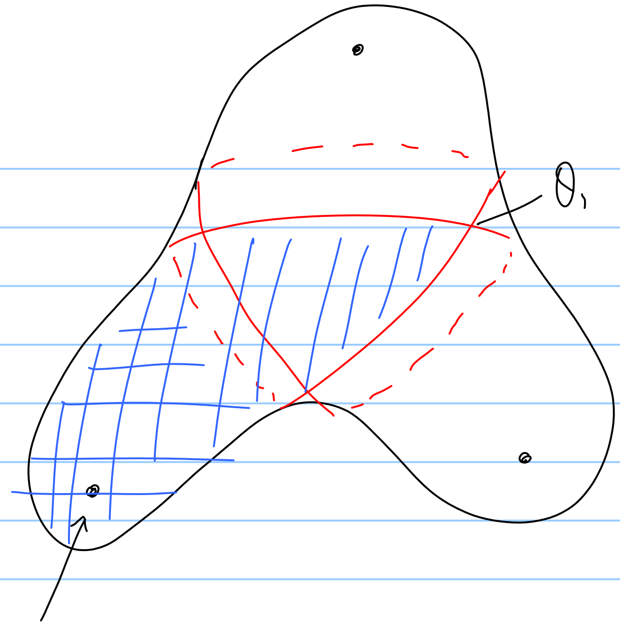
$(p,q) \in S^{n-2} \times S^{n-2}$

$i(p) = i(q)$

$p \neq q$





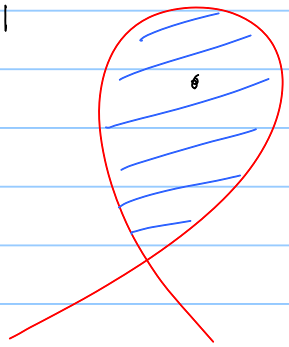


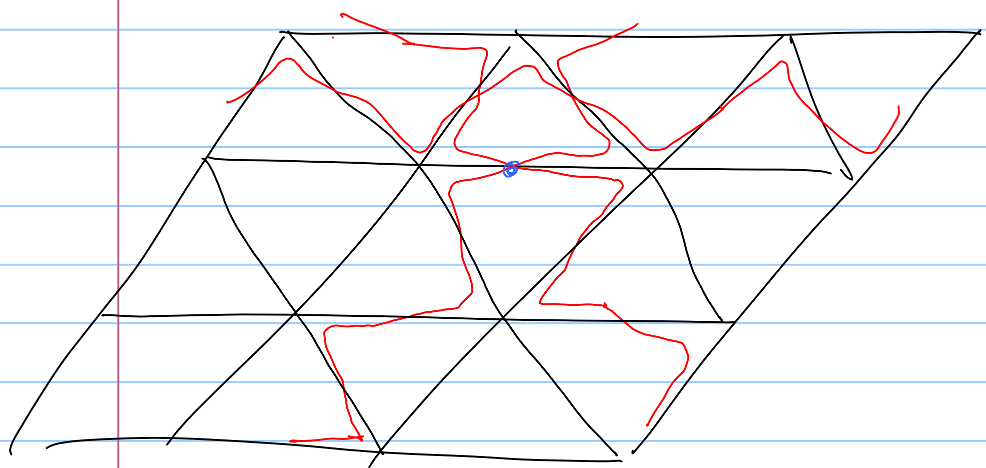
θ_1

+ similar picture on other side

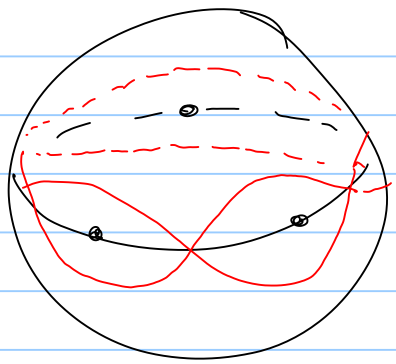
D_1

$\rightsquigarrow \quad \mu'(\theta_1) = \pm Q \cdot 1$





$$X^3 \longrightarrow \mathbb{CP}^1$$



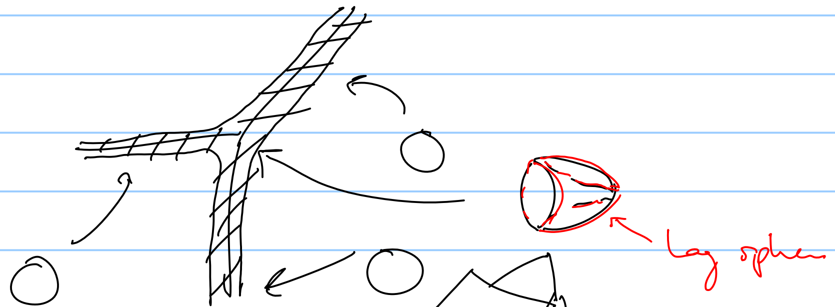
In dim ≥ 2:

$$\begin{array}{ccccc} \mathbb{C}P^{n-2} & = & \{\sum z_i = 0\} & \subset & \mathbb{C}P^{n-1} \\ \cup & & & & \cup \\ D & & & & \{z_1 \cdots z_n = 0\} \end{array}$$

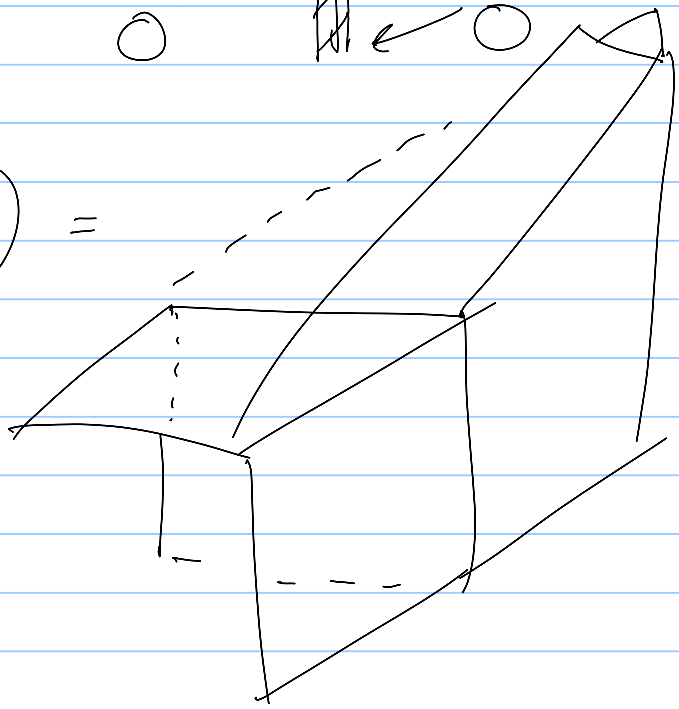
$$\Rightarrow \quad \begin{tikzcd} \mathbb{C}P^{n-2} \setminus D = \{\sum z_i = 0\} \subset (\mathbb{C}^\times)^{n-1} \arrow[d, "\mathrm{Arg}"'] \arrow[dr, "\mathrm{Log}"] & \\ (S^1)^{n-1} & \mathbb{R}^{n-1} \end{tikzcd}$$

$$\mathbb{C}P^{n-2} \setminus D = \ddot{P}^{n-2}$$

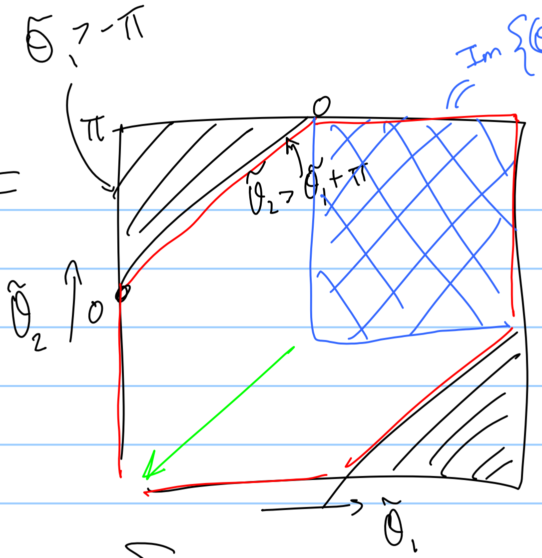
$\mathrm{Log}(P^1) =$



$\mathrm{Log}(P^2) =$



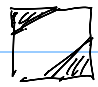
$$\mathrm{Arg}(\mathcal{P}') =$$



$\tilde\theta_1 = \theta_1 - \theta_3$

$\tilde\theta_2 = \theta_2 - \theta_3$

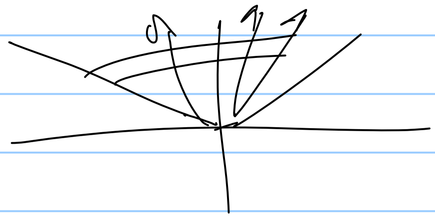
moving θ_3 to arbitrary value $\in [0,\pi]$ eliminates rest of white in



$$= \{(\theta_1,\theta_2,\theta_3) : \sum e^{r_j + i\theta_j} = 0 \text{ for some } r_j\} \Big/ S^1$$

because in projective space

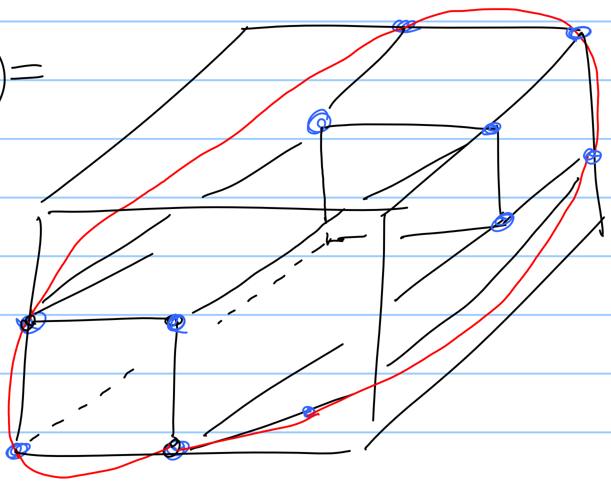
$$= \{(\theta_1,\theta_2,\theta_3) : \text{not contained in some cone}$$



$$\}$$

$$= (S^1)^3 / S^1 \setminus \underbrace{\mathrm{im}([0,\pi]^3)}_{\text{can use } S^1 \text{ action to rotate to this one cone}}$$

$$\mathrm{Arg}(\mathcal{P}^2) =$$



vertices = self-intersections

$$= \{(\theta_1,\dots,\theta_4) : \theta_i \in \{0,\pi\}, \text{ except } (0,0,0,0), (\pi,\pi,\pi,\pi)\}$$

$$\updownarrow$$

generators of $\Lambda^* \mathbb{C}^4$, except 1 and $\theta_1\theta_2\theta_3\theta_4$

We construct

$$L : S^{n-2} \hookrightarrow \mathbb{CP}^{n-2} \setminus D$$ as follows:

$$\begin{tikzcd}
S^{n-2} = \Gamma(\varepsilon df) \arrow[rdd, bend right] & \subset D^*_\varepsilon S^{n-2} \arrow[d] \\
& D^*_\varepsilon \mathbb{RP}^{n-2} \arrow[d, hook] \\
& \mathbb{CP}^{n-2}
\end{tikzcd}$$

If $Df \pitchfork D_j^{\mathbb{R}}$ $\forall j$

then this image of $\Gamma(\varepsilon df)$ will avoid D_j $\forall j$

Now $S^{n-2} = \{(x_1, \dots, x_n) : \sum x_i^2 = 1, \sum x_i = 0\}$

$\downarrow$

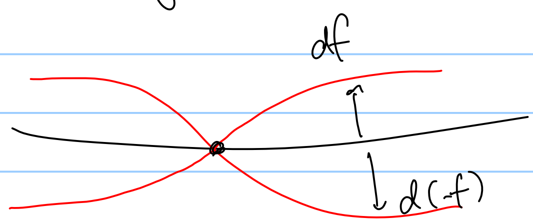
$\mathbb{R}P^{n-2} = \{[x_1 : \dots : x_n] : \sum x_i = 0\}$

Define $f := \sum g(x_j)$,

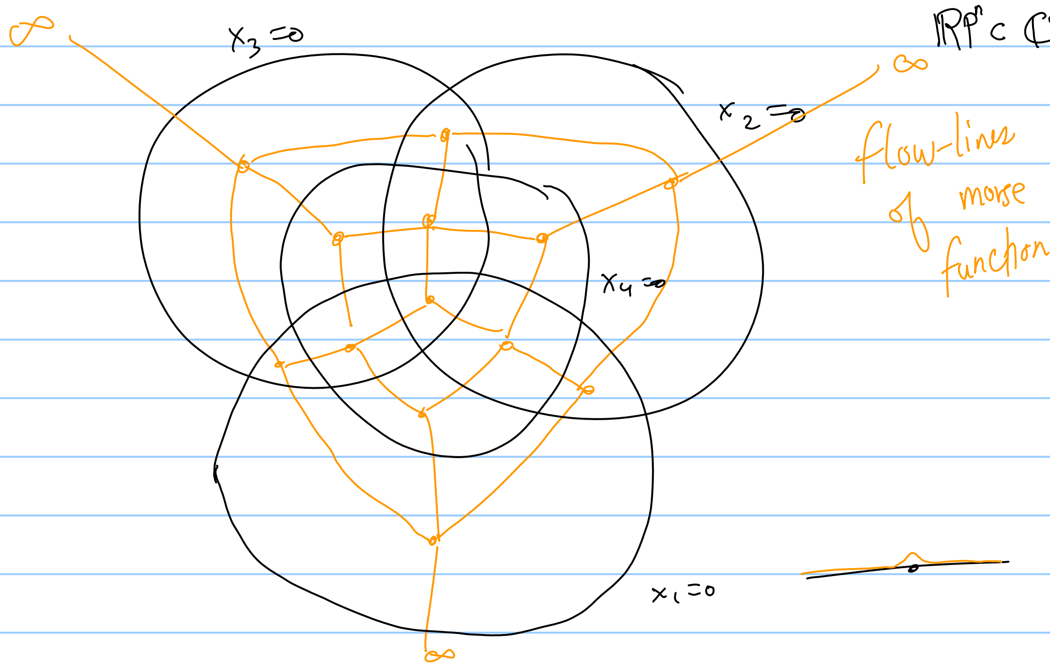
where

$f(x) = -f(-x)$

$\Rightarrow$ self-intersections = crit. points



P^2 stereographic projection of double cover S^n of $\mathbb{R}P^n \subset \mathbb{C}P^n$



Crit pts $\longleftrightarrow$ regions in complement of $D_j^{\mathbb{R}}$

$$\| $$

$$\left\{ (x_1,\dots,x_n) : \begin{array}{l} x_i > 0 \text{ for } i \in K \\ x_i < 0 \text{ for } i \notin K \\ \text{where } K \subset \{1,\dots,n\} \\ K \neq \phi, \{1,\dots,n\} \end{array} \right\}$$

$\longleftrightarrow$ generators of $\Lambda^{\bullet}\mathbb{C}^n$ except $1, \; \theta_1 \wedge \dots \wedge \theta_n$

Call p_K = gens of $CF^*(L,L)$ corresp to $K \subset \{1,\dots,n\}$.

Compute: $H_1(\mathcal{G}P^{n-2}) \cong \mathbb{Z} \oplus \mathbb{Z}^n \big/ \mathbb{Z}\left((1-n) \oplus (\underset{e_1}{1},\dots,\underset{e_n}{1})\right)$

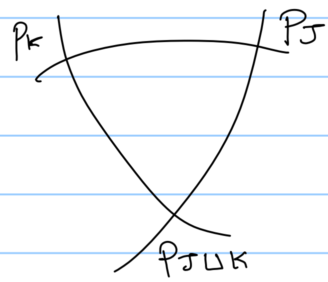
Generator p_K of $CF^*(L,L)$ has degree $(-|K|, e_K)$

$$e_K = \sum_{i \in K} e_i$$

$\Longrightarrow$ the only possible μ^2 send

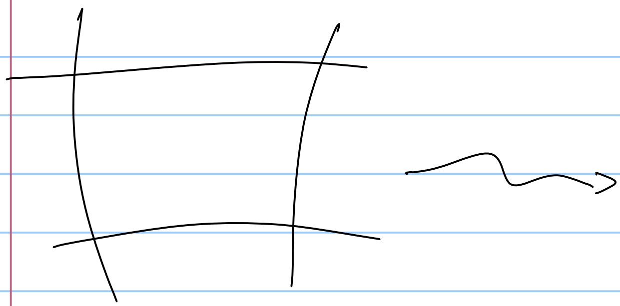
$$p_J \otimes p_K \longmapsto p_{J \cup K}$$

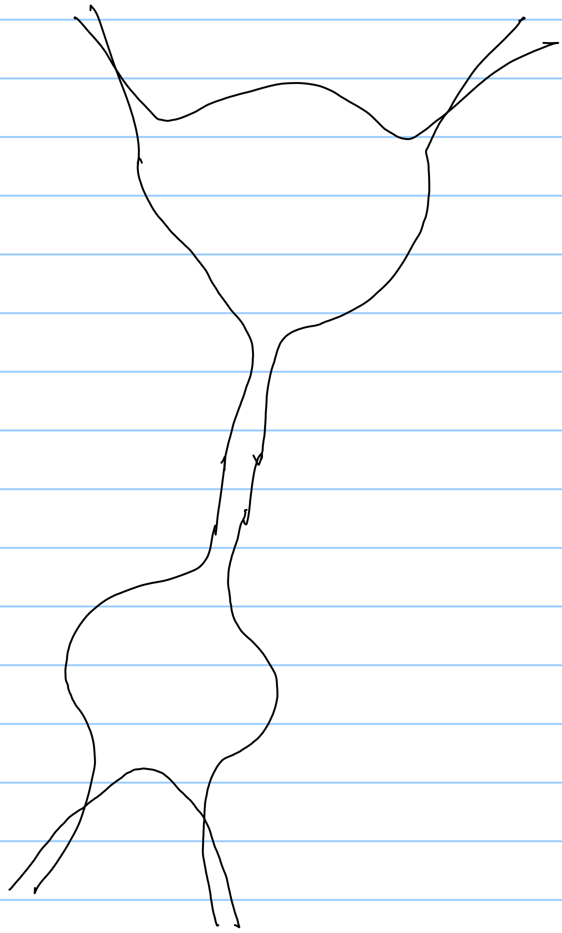
To count triangles

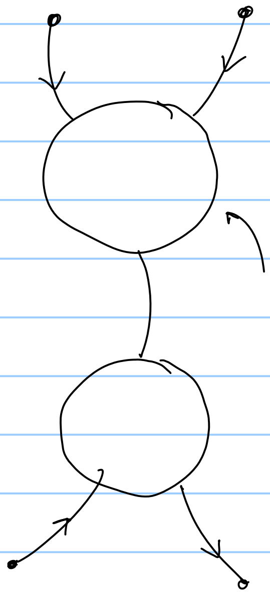


We degenerate $L = \Gamma(\varepsilon df)$

by sending $\varepsilon \to 0$.

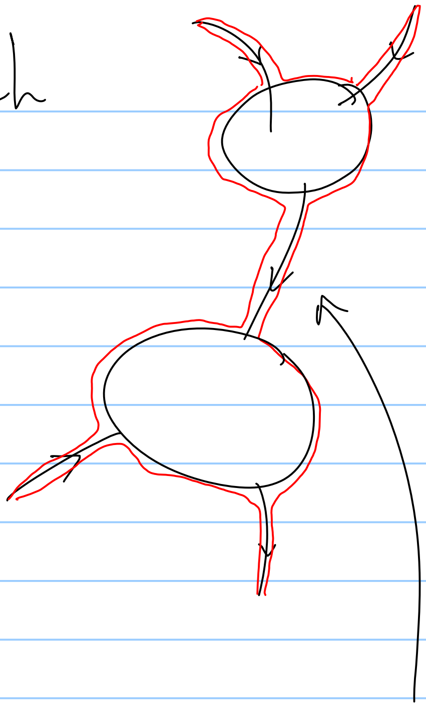






holom disc
w/ ∂ on $\mathbb{RP}^{n-2}$
$\Rightarrow$ can double
to real
holom
sphere,
and count.

Each



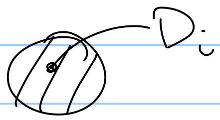
must come with a lift of its boundary to S^{n-2}, switching sheets at crit points of f.

opp sheets: flowline of ∇f

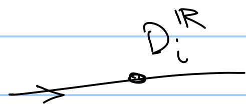
same sheets: flowline of ∇h where $h: S^{n-2} \longrightarrow \mathbb{R}$ Morse function with two crit pts.

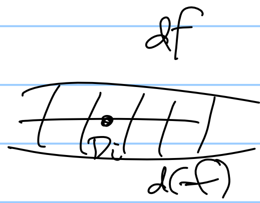
You can define an intersection number of such an object with each D_i

$u \cdot D_i =$ { $+1$ for each



$+1$ for each





$+1$ for