FLOWS ON HOMOGENEOUS SPACES - COURSE OUTLINE

ELON LINDENSTRAUSS

SYLLABUS

The space of lattices in $\mathbb{R}^n$.
(0) Lattices and fundamental domains in locally compact groups.
(1) Identification of space of lattices in $\mathbb{R}^n$ with $\text{SL}(n,\mathbb{Z}) \backslash \text{SL}(n,\mathbb{R})$.
(2) Construction of Siegel domains.
(3) Mahler’s compactness criterion.
(4) $\text{SL}(n,\mathbb{Z})$ is a lattice!

Ergodicity and mixing properties of flows on homogeneous spaces.
(1) Review of ergodicity and mixing in measure preserving systems.
(2) Unitary representations and the Mautner phenomenon
(3) Howe-Moore Theorem

Divergence properties of unipotent flows.
(1) The Dani-Kleinbock-Margulis Theorem.
(2) Existence of minimal subsets for unipotent flows
(3) Radon measures invariant under unipotent flows

Arithmetic lattices in locally compact groups.
(1) Groups defined over $\mathbb{Q}$.
(2) Some cases of Borel Harish-Chandra theorem.
(3) Borel Density Theorem

The Oppenheim Conjecture.
(1) Some basics about quadratic forms and statement of the Oppenheim Conjecture
(2) Translation to dynamics
(3) Proof of the Oppenheim Conjecture

Action of full diagonal group.
(1) A conjecture of Cassels-Swinnerton-Dyer
(2) Translation to dynamics
(3) Classification of divergent orbits
(4) Topological entropy and the action of the full diagonal group.

Date: February 21, 2010.
Introduction to Ratner’s measure classification of theorem.

(1) Statement of Ratner’s theorems about measure classification, orbit closures, and distribution of individual orbits.
(2) Proof of some cases of Ratner’s measure classification of theorem as time allows.

Prerequisites

Prerequisites will be kept to a minimum. We will pursue an aggressively lowbrow approach to algebraic groups, and with respect to ergodic theory will not use much beyond the pointwise ergodic theorem (certainly Ch. 2 of [3] contains more than we will use).

Selection of related books and papers