

We proved in [1] that in a game of perfect information, the set $H(M)$ of payoff vectors to mixed strategy n -tuples is the same as the set $H(C)$ of payoff vectors to correlated strategy n -tuples. We also conjectured that for games without chance moves, this property is characteristic of game structures¹ of perfect information, in the sense that if a game structure enjoys this property for each choice of the payoff function, then it is equivalent to a game structure of perfect information. It is the object of this note to verify this conjecture.

Since the one direction is already established, we need only show that if Γ is a game structure without chance that is not equivalent to one of perfect information, then it is possible to attach a payoff function h to Γ such that $H(M) \neq H(C)$. First, we may suppose without loss of generality that Γ has at least two players and is completely inflated [2]. Now let V be an information set that contains more than one member, such that the number of alternatives at moves of V is at least 2; let V belong to player i . Let x_1 and x_2 be distinct moves in V , and let y be the maximum move such that $y < x_1$ and $y < x_2$. If y belongs to player i , then x_1 is a member of a subset B of V that does not contain x_2 and that is isolated in V ; hence Γ is not completely inflated [2, p. 226], contrary to our assumption. Therefore y belongs to a player j other than i . Now let us choose four plays w_{mn} for $m, n = 1, 2$ so that w_{mn} passes through x_m and chooses the n 'th alternative there; apart from these conditions the w_{mn} may be chosen arbitrarily. We define the payoff function h by² $h_1(w_{11}) = h_2(w_{22}) = 4$, all the other payoffs being 0. Then it is easily seen that the payoff vector $(2, 2, 0, \dots, 0)$ is in $H(C)$ but not in $H(M)$.

In the general case, when Γ may have chance moves, we have the following theorem: *A necessary and sufficient condition that $H(M) = H(C)$ for all payoff functions h and all probability distributions at the chance moves of Γ is that Γ be of effectively perfect information* (cf. [2, p. 232]). Again, the sufficiency follows easily from Theorem 9.1 of [1]. To prove the necessity, suppose that Γ is not of effectively perfect information. As before, we may suppose without loss of generality that Γ

¹ A *game structure* [2] is the same as an extensive game, but without the payoff function h on the set of all plays (or endpoints), and without the probability distributions p at the chance moves. The latter condition is of course vacuous when we are referring to games without chance.

² What is denoted h_i here was denoted μ^i in [1].

is completely inflated. Since Γ is not of effectively perfect information, there are information sets V and U belonging to different players i and j respectively, such that V contains two moves x_1 and x_2 , and there are different indices μ and ν for which $x_1 \in U_\mu$ and $x_2 \in U_\nu$. Define y , the w_{mn} , and h as in the previous proof. If y is a personal move then the remainder of the proof is as in the previous proof. If y is a chance move, let the probability of the alternatives lying on the "subplays" w_{x_1} and w_{x_2} be $\frac{1}{2}$ each, the other probabilities being 0. Then $(1,1,0,\dots,0)$ is in $H(C)$ but not in $H(M)$. (In either case—whether y is a personal move or a chance move—the probability distributions at chance moves (other than y) lying on the w_{mn} should be chosen so that the probability of choosing the alternatives on the w_{mn} is 1.)

To illustrate this proof, we give in Figure 1 a simple example of a game structure that is not of effectively perfect information, and construct h and p in accordance with the above to show that $H(C) \neq H(M)$.

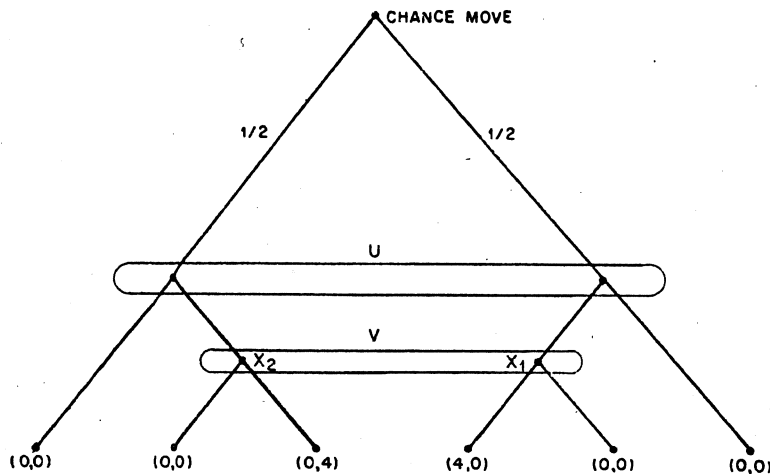


Figure 1

Like Dalkey's criteria [2], the criteria given here characterize the notion of perfect information in terms of the strategy spaces and payoffs—the normal forms—of the games in question. Unlike Dalkey's criteria, though, the present criteria make no use of any particular "solution" notion, such as that of Nash equilibrium point. In form our characterization is more like the characterization of games of perfect recall given by Kuhn in [3] (Theorem 4, p. 214).

REFERENCES

1. AUMANN, R. J., 1960, Acceptable points in games of perfect information, *Pac. Jour. of Math.* 10, 381–417 [Chapter 21].
2. DALKEY, N., 1953, Equivalence of information patterns and essentially determinate games. Contributions to the Theory of Games II, *Ann. of Math. Study* 28, Princeton Univ. Press, 217–243.
3. KUHN, H. W., Extensive games and the problem of information, *ibid.*, 193–216.

³ U_μ is the set of vertices x of the game tree such that the unique unicursal path (or "subplay") w_x from the first move of the game to x passes through an alternative at a move in U with index μ (2, p. 226).